



## INVESTIGATION OF VIBRATIONS OF IDEALLY LUBRICATED CIRCULAR ELEMENT OF AGRICULTURAL MACHINES

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### Abstract

Circular structures are used as elements of agricultural machines. Investigation of their vibrations is an important engineering problem. Here a simplified model based on two basic assumptions is being investigated. The first assumption is that in the normal direction the internal and external circular surfaces of the structure are fastened, that is the corresponding displacements are assumed equal to zero. The second assumption is that of ideal lubrication of both circular surfaces, that is it is assumed that there is no force of friction in the tangential direction resisting to the motion of the structure and thus the tangential displacements are free to perform their motions. Of course, those assumptions are not exactly observed in practical engineering applications, but they enable us to determine the eigenmodes of the investigated circular element of agricultural machines. The presented approach serves as an initial approximation for investigation of the described engineering problem especially having in mind the assumption of ideal lubrication.

**Keywords:** agricultural machines, circular structure, ideal lubrication, vibrations, eigenmodes.

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### 1. Introduction

Circular structures are used as elements of agricultural machines. Investigation of their vibrations is an important engineering problem.

Here a simplified model based on two basic assumptions is being investigated.

1) The first assumption is that in the normal direction the internal and external circular surfaces of the structure are fastened, that is the corresponding displacements are assumed equal to zero.

2) The second assumption is that of ideal lubrication of both circular surfaces, that is it is assumed that there is no force of friction in the tangential direction resisting to the motion of the structure and thus the tangential displacements are free to perform their motions.

Of course, those assumptions are not exactly observed in practical engineering applications, but they enable us to determine the eigenmodes of the investigated circular element of agricultural machines.

The presented approach serves as an initial approximation for investigation of the described engineering problem especially having in mind the assumption of ideal lubrication.

The investigation presented here is based on the material presented in (Zienkiewicz, 1975) and (Bathe, Wilson, 1982). Vibrations and their analysis are investigated in (Blekhman, 2018). Robots and their elements are analyzed in (Glazunov, 2018). Transmissions and their dynamics are presented in (Kurila, Ragulskienė, 1986). Vibrations in essentially nonlinear systems are investigated in (Ragulskienė, 1974).

First the model of the investigated circular element of agricultural machines is described. The two-dimensional problem of plane stress with the change of both directions of nodal displacements into the

tangential and normal ones is solved. Then eigenmodes of the investigated system are obtained and presented graphically.

## 2. Model of the ideally lubricated circular element of agricultural machines

Nodal coordinates of the investigated structure are assumed as:

$$x = \left( r_1 + (r_2 - r_1) \frac{j-1}{8} \right) \cos \left( -2\pi \frac{i-1}{64} \right), \quad (1)$$

$$y = \left( r_1 + (r_2 - r_1) \frac{j-1}{8} \right) \sin \left( -2\pi \frac{i-1}{64} \right), \quad (2)$$

where  $r_1$  is the internal radius of the circular structure,  $r_2$  is the external radius of the circular structure and:

$$i = 1, \dots \quad (3)$$

$$j = 1, \dots \quad (4)$$

Tangential vector is given as:

$$\{t\} = \begin{Bmatrix} t_1 \\ t_2 \end{Bmatrix} = \begin{Bmatrix} \sin \left( -2\pi \frac{i-1}{64} \right) \\ -\cos \left( -2\pi \frac{i-1}{64} \right) \end{Bmatrix}. \quad (5)$$

Normal vector is given as:

$$\{n\} = \begin{Bmatrix} n_1 \\ n_2 \end{Bmatrix} = \begin{Bmatrix} \cos \left( -2\pi \frac{i-1}{64} \right) \\ \sin \left( -2\pi \frac{i-1}{64} \right) \end{Bmatrix}. \quad (6)$$

Transformation matrix for the node  $k$  has the form:

$$[T_k] = \begin{bmatrix} t_1 & n_1 \\ t_2 & n_2 \end{bmatrix}. \quad (7)$$

Nodal displacements of the finite element are transformed as:

$$\{\delta_{xy}\} = [T]\{\delta_m\}, \quad (8)$$

where  $\{\delta_{xy}\}$  is the vector of nodal displacements in the global directions,  $\{\delta_m\}$  is the vector of nodal displacements in the local directions and:

$$[T] = \begin{bmatrix} [T_1] & [0] & \dots & & & \\ [0] & [T_2] & \dots & & & \\ \vdots & \vdots & \ddots & \ddots & & \\ [0] & [0] & \dots & & & \end{bmatrix} \quad (9)$$

where it is assumed that the Lagrange quadratic finite element with nine nodes is used.

The stiffness matrix in the local directions has the form:

$$[\bar{K}] = [T]^T [K] [T], \quad (10)$$

where  $[K]$  is the usual stiffness matrix of the problem of plane stress.

The mass matrix in the local directions has the form:

$$[\bar{M}] = [T]^T [M] [T], \quad (11)$$

where  $[M]$  is the usual mass matrix of the problem of plane stress.

## 3. Investigation of the ideally lubricated circular element of agricultural machines

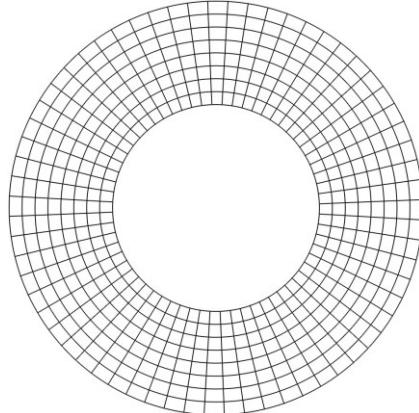
Normal displacements on the internal and external radii of the circular structure are assumed to be equal to zero.

The following parameters of the investigated structure are assumed:

$$r_1 = 0.2, r_2 = 0.4, E = 6 \cdot 10^8, \nu = 0.3, h = 0.01, \rho = 785, \quad (12)$$

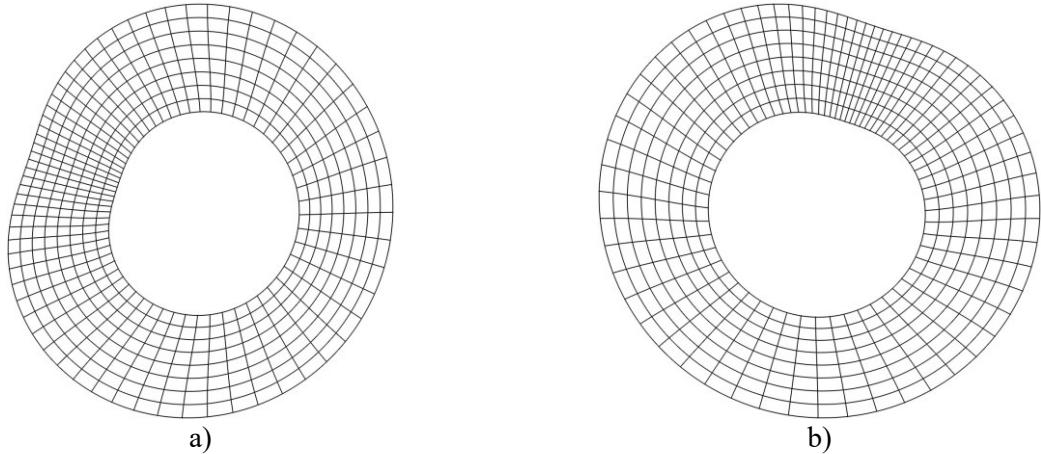
where  $E$  is the modulus of elasticity,  $\nu$  is the Poisson's ratio,  $h$  is the thickness of the structure and  $\rho$  is the density of the material of the structure.

The first eigenmode corresponding to rigid body motion of the structure is presented in Fig. 1.



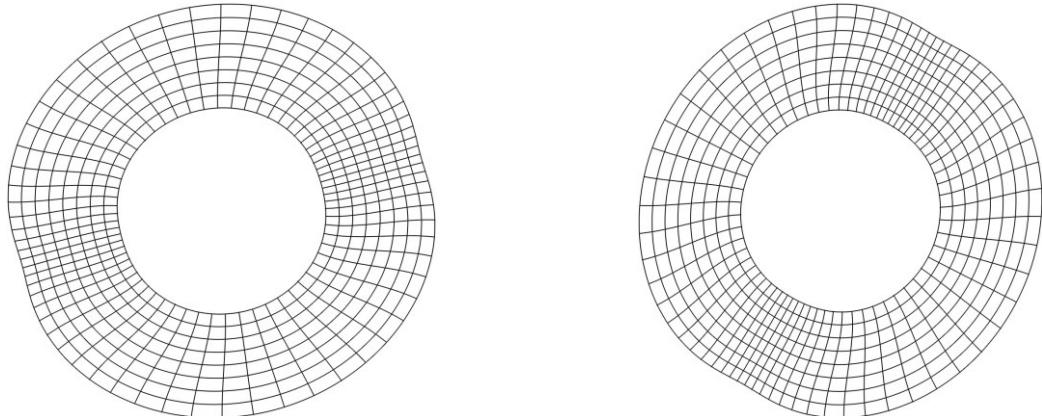
**Fig. 1.** The first eigenmode of the ideally lubricated circular element

The second and third multiple eigenmodes are presented in Fig. 2.



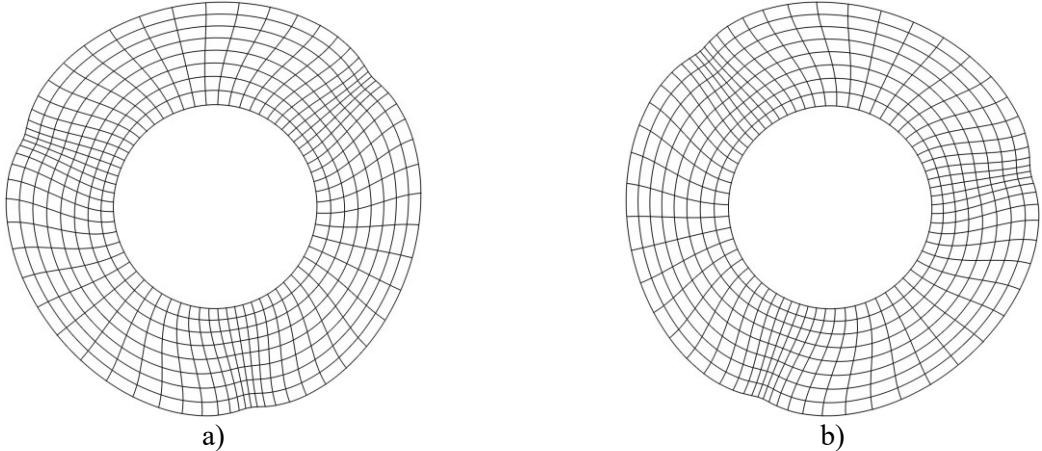
**Fig. 2.** The second and third eigenmodes of the ideally lubricated circular element

The fourth and fifth multiple eigenmodes are presented in Fig. 3.



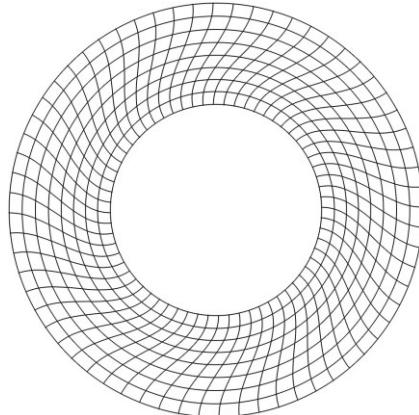
a) b)  
**Fig. 3.** The fourth and fifth eigenmodes of the ideally lubricated circular element

The sixth and seventh multiple eigenmodes are presented in Fig. 4.



**Fig. 4.** The sixth and seventh eigenmodes of the ideally lubricated circular element

The eighth eigenmode is presented in Fig. 5.



**Fig. 5.** The eighth eigenmode of the ideally lubricated circular element

The main inaccuracy of the presentation is in the fact that by representing tangential displacements as rather large ones they produce small displacements in the radial direction.

#### 4. Conclusions

Circular structures are used as elements of agricultural machines. Investigation of their vibrations is an important engineering problem. Here a simplified model is being investigated.

It is assumed that in the normal direction the internal and external circular surfaces of the structure are fastened, that is the corresponding displacements are assumed equal to zero. It is also assumed that both circular surfaces are ideally lubricated and there is no force of friction in the tangential direction.

The two-dimensional problem of plane stress with the change of both directions of nodal displacements into the tangential and normal ones is solved. Eigenmodes of the investigated circular element of agricultural machines are determined and investigated. As seen from the obtained results the main inaccuracy of the presentation is in the fact that by representing tangential displacements as rather large ones they produce small displacements in the radial direction.

The presented approach serves as an initial approximation for investigation of the described engineering problem especially having in mind the assumption of ideal lubrication.

## References

- [1] K. J. Bathe; E. L. Wilson, 1982. *Численные методы анализа и метод конечных элементов. (Numerical Methods in Finite Element Analysis)*. Moscow: Stroiizdat. P. 448.
- [2] I. I. Blekhman, 2018. *Вибрационная механика и вибрационная реология (теория и приложения). (Vibration Mechanics and Vibration Reology (Theory and Applications))*. Moscow: Physmathlit. P. 752.
- [3] V. Glazunov, 2018. *Новые механизмы в современной робототехнике. (New Mechanisms in Contemporary Robot Engineering)*. Moscow: Tehnosphere. P. 316.
- [4] R. Kurila; V. Ragulskienė, 1986. *Двумерные вибрационные приводы. (Two – Dimensional Vibro – Transmissions)*. Vilnius: Mokslas. P. 137.
- [5] V. Ragulskienė, 1974. *Виброударные системы. (Vibro-Shock Systems)*. Vilnius: Mintis. P. 320.
- [6] O. C. Zienkiewicz, 1975. *Метод конечных элементов в технике. (The Finite Element Method in Engineering Science)*. Moscow: Mir. P. 544.

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