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INVESTIGATION OF VIBRATIONS OF LAYERED ELEMENTS OF AGRICULTURAL MACHINES

***Jūratė Ragulskienė*[1], *Arvydas Pauliukas*[2], *Petras Paškevičius*[3], *Rimas Maskeliūnas*[4], *Vytautas Maskeliūnas*[5], *Anatolii Korpach*[6], *Liutauras Ragulskis*[7]**
[1]Kaunas University of Technology, Kaunas, Lithuania
[2]Vytautas Magnus University, Akademija, Kaunas District, Lithuania
[3]Company "Vaivora", Kaunas, Lithuania
[4, 5]Vilnius Gediminas Technical University, Vilnius, Lithuania
[6]National Transport University, Kyiv, Ukraine
[7]Vytautas Magnus University, Kaunas, Lithuania

Abstract

Layered materials are used as cases of some types of agricultural machines and devices. They have an advantage in the transmission and suppression of some types of vibrations in the process of operation of those machines. In this paper a layered beam type structure is investigated. It is assumed that layers are of two types: 1) of the beam type and 2) of the elastic body type. Usually, the lower and upper layers are of beam type and the internal layer is of elastic body type. Finite element models of the layer of beam type as well as of the layer of elastic body type are developed and described in the paper. Based on them finite element of a layered beam is obtained. Eigenmodes of the beam of this type are calculated and investigated. The presented results are applicable in the process of design of elements of agricultural machines and other engineering devices.
Keywords: *Layered material, agricultural machines, case of the machine, finite elements, eigenmodes.*

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1. Introduction

Layered materials are used as cases of some types of agricultural machines and devices. They have an advantage in the transmission and suppression of some types of vibrations in the process of operation of those machines.

In this paper a layered beam type structure is investigated. It is assumed that layers are of two types: 1) of the beam type and 2) of the elastic body type. Usually, the lower and upper layers are of beam type and the internal layer is of elastic body type.

Finite element models of the layer of beam type as well as of the layer of elastic body type are developed and described in detail in this paper. Based on them finite element of a layered beam having the lower and upper layers of beam type and the internal layer of elastic body type is obtained. Eigenmodes of the beam of this type are calculated and investigated.

The presented results are applicable in the process of design of elements of agricultural machines and other engineering devices.

Basic models for analysis of elastic structures by using the method of finite elements are presented in (Zienkiewicz, 1975) and (Bathe, Wilson, 1982). Vibrations of beams are investigated in (Pany, 2023), (Pany, Rao, 2002), (Pany, Rao, 2004) and in many other related papers. Theoretical analysis of vibrations in mechanical engineering is presented in (Blekhman, 2018). Mechanisms having beams as their elements in the structures of robots are described in (Glazunov, 2018). Vibrations of transmissions are investigated in (Kurila, Ragulskienė, 1986). Essentially nonlinear vibrations are analyzed in (Ragulskienė, 1974).

First the model of layer of beam type is described in detail, then the model of layer of elastic body type is described in detail. Finally, the finite element of a layered beam with external layers of beam type and internal layer of elastic body type is obtained. Based on the presented material eigenmodes of the layered beam are represented graphically and described.

2. Model of the layer of beam type

Three nodal parameters are assumed: displacement in the direction of the y axis v_{12}, displacement of the lower surface in the direction of the x axis u_1 and displacement of the upper surface in the direction of the x axis u_2.

Displacement of the middle line of the layer in the direction of the x axis u and displacement of the middle line of the layer in the direction of the y axis v are expressed as:

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} [N_u] \\ [N_v] \end{bmatrix} \{\delta\}, \tag{1}$$

where $\{\delta\}$ is the vector of nodal displacements and:

$$\begin{bmatrix} [N_u] \\ [N_v] \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2}N_1 & \frac{1}{2}N_1 & \cdots \\ N_1 & 0 & 0 & \cdots \end{bmatrix}, \tag{2}$$

where N_1, … are shape functions of the finite element.

Rotation of the middle line of the layer θ is expressed as:

$$\theta = [N_\theta]\{\delta\}, \tag{3}$$

where:

$$[N_\theta] = \begin{bmatrix} 0 & \frac{1}{2b}N_1 & -\frac{1}{2b}N_1 & \cdots \end{bmatrix} \tag{4}$$

where b is semi thickness of the layer.

If the structure experiences static displacements, then the longitudinal stress multiplied by the cross-sectional area of the layer M_σ is calculated:

$$M_\sigma = E4ab\left[\frac{dN_u}{dx}\right]\{\delta\}, \tag{5}$$

where E is the modulus of elasticity of the layer, a is the semi width of the layer and $\{\delta\}$ is the vector of nodal static displacements.

Then the stiffness matrix of the layer has the form:

$$[K] = \int \left(\begin{aligned} &\left[\frac{dN_u}{dx}\right]^T E4ab\left[\frac{dN_u}{dx}\right] + \left[\frac{dN_\theta}{dx}\right]^T E\frac{4ab^3}{3}\left[\frac{dN_\theta}{dx}\right] + \\ &+\left[[N_\theta] - \left[\frac{dN_v}{dx}\right]\right]^T \frac{G4ab}{1.2}\left[[N_\theta] - \left[\frac{dN_v}{dx}\right]\right] + \left[\frac{dN_v}{dx}\right]^T M_\sigma \left[\frac{dN_v}{dx}\right] \end{aligned} \right) dx, \tag{6}$$

where $G = \dfrac{E}{2(1+\nu)}$ is the shear modulus of the layer and ν is the Poisson's ratio of the layer.

The mass matrix of the layer has the form:

$$[M] = \int \left([N_u]^T \rho 4ab [N_u] + [N_v]^T \rho 4ab [N_v] + [N_\theta]^T \rho \frac{4ab^3}{3} [N_\theta] \right) dx, \tag{7}$$

where ρ is the density of the material of the layer.

3. Model of the layer of elastic body type

Four nodal parameters are assumed: displacement of the lower surface in the direction of the x axis u_1, displacement of the lower surface in the direction of the y axis v_1, displacement of the upper surface in the direction of the x axis u_2 and displacement of the upper surface in the direction of the y axis v_2.

Displacement in the direction of the x axis u and displacement in the direction of the y axis v are expressed as:

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \frac{2b-y}{2b}\begin{Bmatrix} \bar{u} \\ \bar{v} \end{Bmatrix} + \frac{y}{2b}\begin{Bmatrix} \bar{\bar{u}} \\ \bar{\bar{v}} \end{Bmatrix}, \tag{8}$$

where b is semi thickness of the layer, upper dash denotes quantities on the lower surface of the layer and double upper dash denotes quantities on the upper surface of the layer. Thus:

$$\begin{Bmatrix} \bar{u} \\ \bar{v} \end{Bmatrix} = \left[\bar{N}\right]\{\delta\}, \tag{9}$$

where $\{\delta\}$ is the vector of nodal displacements and:

$$\left[\bar{N}\right] = \begin{bmatrix} \left[\bar{N}_u\right] \\ \left[\bar{N}_v\right] \end{bmatrix} = \begin{bmatrix} N_1 & 0 & 0 & 0 & \dots \\ 0 & N_1 & 0 & 0 & \dots \end{bmatrix}. \tag{10}$$

where N_1, … are shape functions of the finite element. Also:

$$\begin{Bmatrix} \bar{\bar{u}} \\ \bar{\bar{v}} \end{Bmatrix} = \left[\bar{\bar{N}}\right]\{\delta\}, \tag{11}$$

where:

$$\left[\bar{\bar{N}}\right] = \begin{bmatrix} \left[\bar{\bar{N}}_u\right] \\ \left[\bar{\bar{N}}_v\right] \end{bmatrix} = \begin{bmatrix} 0 & 0 & N_1 & 0 & \dots \\ 0 & 0 & 0 & N_1 & \dots \end{bmatrix}. \tag{12}$$

Strains are expressed as:

$$\{\varepsilon\} = \frac{2b-y}{2b}\begin{Bmatrix} \frac{d\bar{u}}{dx} \\ 0 \\ \frac{d\bar{v}}{dx} \end{Bmatrix} + \frac{y}{2b}\begin{Bmatrix} \frac{d\bar{\bar{u}}}{dx} \\ 0 \\ \frac{d\bar{\bar{v}}}{dx} \end{Bmatrix} + \frac{1}{2b}\begin{Bmatrix} 0 \\ \bar{\bar{v}} - \bar{v} \\ \bar{\bar{u}} - \bar{u} \end{Bmatrix} = \left(\frac{2b-y}{2b}\left[\bar{B}\right] + \frac{y}{2b}\left[\bar{\bar{B}}\right] + \frac{1}{2b}\left[B\right]\right)\{\delta\}, \tag{13}$$

where:

$$\left[\bar{B}\right] = \begin{bmatrix} \left[\frac{d\bar{N}_u}{dx}\right] \\ [0] \\ \left[\frac{d\bar{N}_v}{dx}\right] \end{bmatrix}, \tag{14}$$

$$\left[\bar{\bar{B}}\right] = \begin{bmatrix} \left[\frac{d\bar{\bar{N}}_u}{dx}\right] \\ [0] \\ \left[\frac{d\bar{\bar{N}}_v}{dx}\right] \end{bmatrix}, \tag{15}$$

$$[B]=\begin{bmatrix}[0]\\ \left[\bar{\bar{N}}_v\right]-\left[\bar{N}_v\right]\\ \left[\bar{\bar{N}}_u\right]-\left[\bar{N}_u\right]\end{bmatrix}. \tag{16}$$

If the structure experiences static displacements, then supplementary stiffness from them is calculated. Thus, the quantity $\frac{\partial v}{\partial x}$ is expressed as:

$$\frac{\partial v}{\partial x}=\frac{2b-y}{2b}\frac{d\bar{v}}{dx}+\frac{y}{2b}\frac{d\bar{\bar{v}}}{dx}=\left(\frac{2b-y}{2b}\left[\bar{G}\right]+\frac{y}{2b}\left[\bar{\bar{G}}\right]\right)\{\delta\}, \tag{17}$$

where $\{\delta\}$ is the vector of nodal static displacements and:

$$\left[\bar{G}\right]=\left[\frac{d\bar{N}_v}{dx}\right], \tag{18}$$

$$\left[\bar{\bar{G}}\right]=\left[\frac{d\bar{\bar{N}}_v}{dx}\right]. \tag{19}$$

Strains are calculated as:

$$\{\varepsilon\}=\left(\frac{1}{2b}[B]+\frac{1}{2}\left[\bar{B}\right]+\frac{1}{2}\left[\bar{\bar{B}}\right]\right)\{\delta\}. \tag{20}$$

Matrix of elastic constants is assumed as:

$$[D]=2a\begin{bmatrix}\frac{E}{1-\nu^2} & \frac{E\nu}{1-\nu^2} & 0\\ \frac{E\nu}{1-\nu^2} & \frac{E}{1-\nu^2} & 0\\ 0 & 0 & \frac{E}{2(1+\nu)1.2}\end{bmatrix}, \tag{21}$$

where a is the semi width of the layer, E is the modulus of elasticity of the layer and ν is the Poisson's ratio of the layer.

Stresses $\{\sigma\}$ multiplied by the width of the layer are calculated as:

$$\{\sigma\}2a=[D]\{\varepsilon\}, \tag{22}$$

and then the quantity M_σ is calculated:

$$M_\sigma=\sigma_x 2a, \tag{23}$$

where σ_x is the longitudinal stress, that is it is the first component of the vector $\{\sigma\}$.

Then the stiffness matrix of the layer has the form:

$$
[K]=\int\begin{pmatrix}
[B]^T[D]\frac{1}{2}[\bar{B}]+[\bar{B}]^T[D]\frac{1}{2}[B]+[B]^T[D]\frac{1}{2}[\bar{\bar{B}}]+[\bar{\bar{B}}]^T[D]\frac{1}{2}[B]+\\
+[\bar{B}]^T[D]\frac{2b}{6}[\bar{\bar{B}}]+[\bar{\bar{B}}]^T[D]\frac{2b}{6}[\bar{B}]+[\bar{B}]^T[D]\frac{2b}{3}[\bar{B}]+[\bar{\bar{B}}]^T[D]\frac{2b}{3}[\bar{\bar{B}}]+\\
+[B]^T[D]\frac{1}{2b}[B]+[\bar{G}]^T M_\sigma\frac{2b}{6}[\bar{\bar{G}}]+[\bar{\bar{G}}]^T M_\sigma\frac{2b}{6}[\bar{G}]+[\bar{G}]^T M_\sigma\frac{2b}{3}[\bar{G}]+\\
+[\bar{\bar{G}}]^T M_\sigma\frac{2b}{3}[\bar{\bar{G}}]
\end{pmatrix}dx. \qquad (24)
$$

The mass matrix of the layer has the form:

$$
[M]=\int\left([\bar{N}]^T\rho 2a\frac{2b}{6}[\bar{\bar{N}}]+[\bar{\bar{N}}]^T\rho 2a\frac{2b}{6}[\bar{N}]+[\bar{N}]^T\rho 2a\frac{2b}{3}[\bar{N}]+[\bar{\bar{N}}]^T\rho 2a\frac{2b}{3}[\bar{\bar{N}}]\right)dx, \qquad (25)
$$

where ρ is the density of the material of the layer.

Here values of the following integrals were used:

$$
\int_0^{2b}\frac{1}{2b}\frac{2b-y}{2b}dy=\int_0^{2b}\frac{1}{2b}\frac{y}{2b}dy=\frac{1}{2}, \qquad (26)
$$

$$
\int_0^{2b}\frac{2b-y}{2b}\frac{y}{2b}dy=\frac{b}{3}, \qquad (27)
$$

$$
\int_0^{2b}\left(\frac{2b-y}{2b}\right)^2dy=\int_0^{2b}\left(\frac{y}{2b}\right)^2dy=\frac{2b}{3}, \qquad (28)
$$

$$
\int_0^{2b}\left(\frac{1}{2b}\right)^2dy=\frac{1}{2b}. \qquad (29)
$$

4. Model of the layered beam

Six nodal parameters are assumed: displacement of the lower beam in the direction of the y axis v_{12}, displacement of the lower surface of the lower beam in the direction of the x axis u_1, displacement of the upper surface of the lower beam in the direction of the x axis u_2, displacement of the upper beam in the direction of the y axis v_{34}, displacement of the lower surface of the upper beam in the direction of the x axis u_3 and displacement of the upper surface of the upper beam in the direction of the x axis u_4.

The matrixes of the previously described layers are added to the corresponding places of the matrixes of the finite element of a layered beam.

5. Investigation of eigenmodes of the layered beam

All displacements on the left end of the layered beam are assumed to be equal to zero. On the right end of the layered beam displacements in the direction of the y axis are assumed to be equal to zero and displacements in the direction of the x axis are assumed to be equal to one. Thus, first the static problem is solved. Then the supplementary stiffness from the static solution is calculated and the eigenmodes are obtained.

The following parameters of the layers of beam type are assumed:

$$
E=6\cdot 10^8,\ \nu=0.3,\ \rho=785,\ a=0.5,\ b=0.1. \qquad (30)
$$

The following parameters of the layer of elastic body type are assumed:

$$
E=0.6\cdot 10^8,\ \nu=0.3,\ \rho=78.5,\ a=0.5,\ b=0.4. \qquad (31)
$$

Eigenmodes are represented on the initial geometry of the structure. The first eigenmode is presented in Fig. 1, the second eigenmode is presented in Fig. 2, …, the eighth eigenmode is presented in Fig. 8.

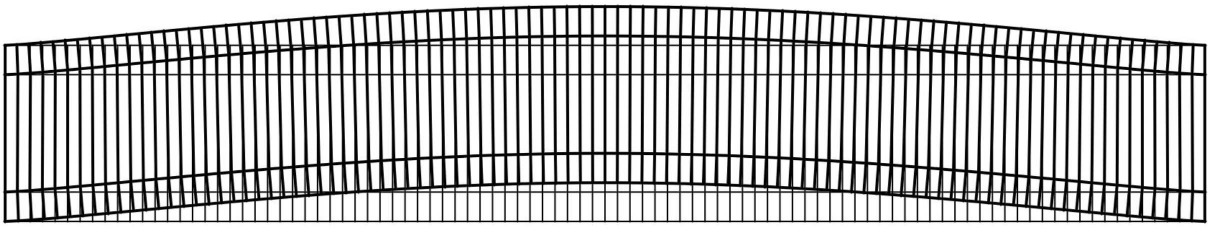

Fig. 1. The first eigenmode of the layered beam

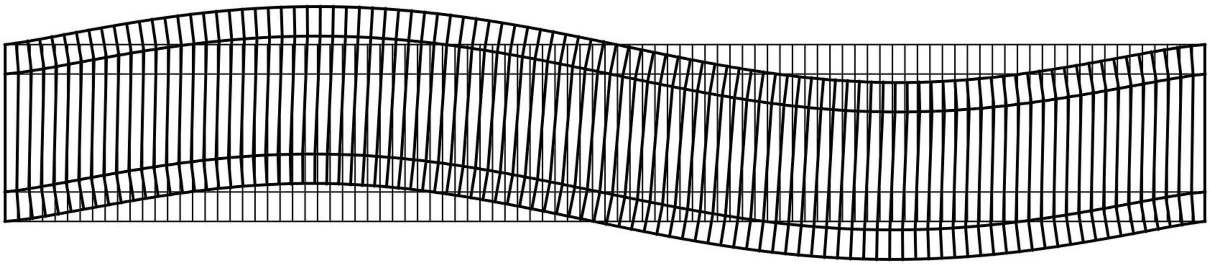

Fig. 2. The second eigenmode of the layered beam

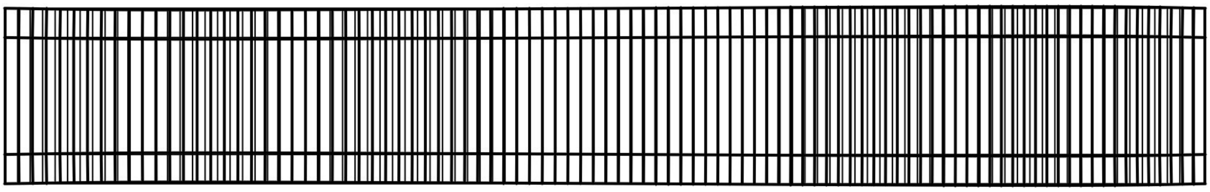

Fig. 3. The third eigenmode of the layered beam

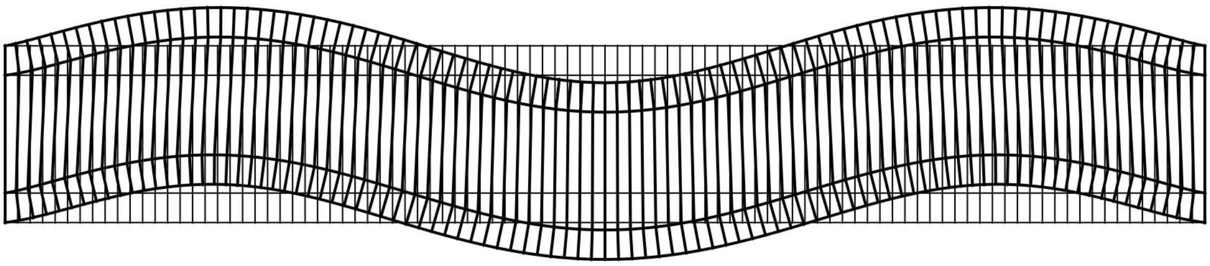

Fig. 4. The fourth eigenmode of the layered beam

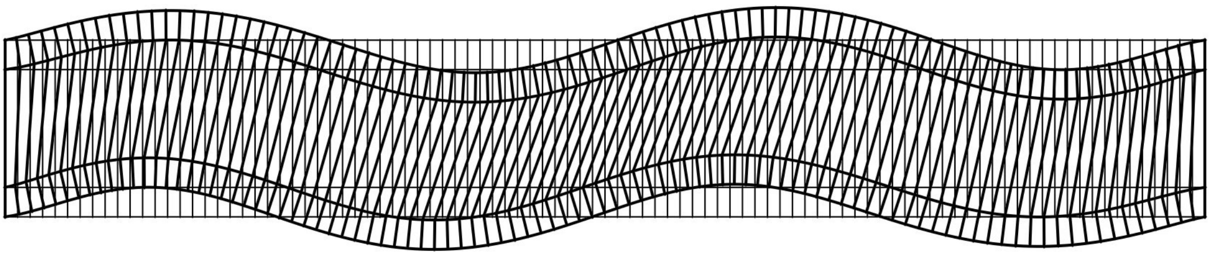

Fig. 5. The fifth eigenmode of the layered beam

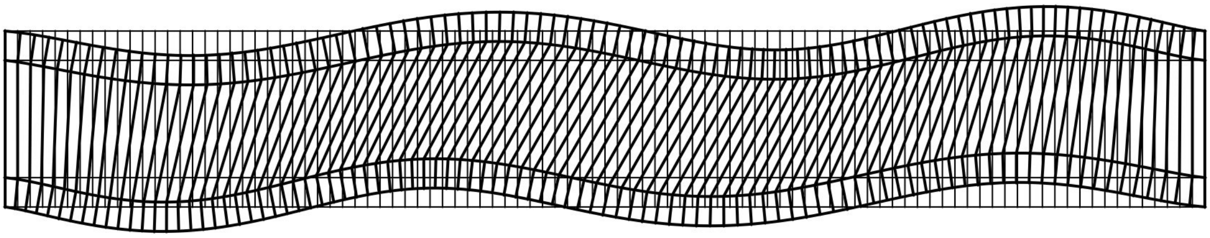

Fig. 6. The sixth eigenmode of the layered beam

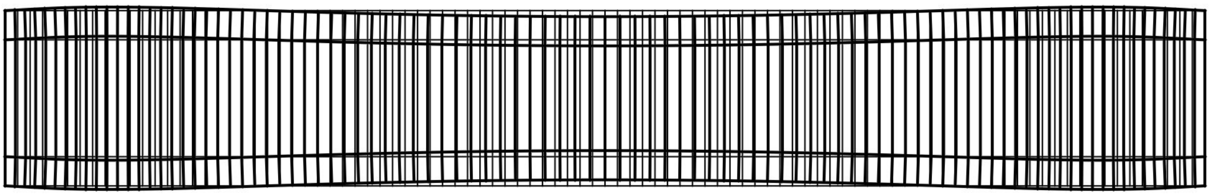

Fig. 7. The seventh eigenmode of the layered beam

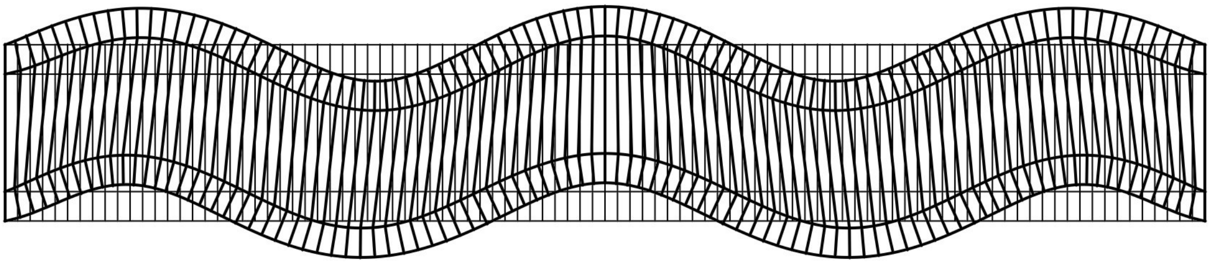

Fig. 8. The eighth eigenmode of the layered beam

From the presented results it is seen that there are eigenmodes of predominantly longitudinal vibrations as well as there are eigenmodes of predominantly transverse vibrations. Substantially different behaviors of the lower and upper layers of beam type and the internal layer of elastic body type are observed.

6. Conclusions

Layered materials are used as cases of some types of agricultural machines and devices. They have an advantage in the transmission and suppression of some types of vibrations in the process of operation of those machines. In this paper a layered beam type structure is investigated. It is assumed that layers are of two types: 1) of the beam type and 2) of the elastic body type.

Finite element models of the layer of beam type as well as of the layer of elastic body type are developed and described. Based on them finite element of a layered beam is obtained. Eigenmodes of the beam of this type are calculated and investigated. From the presented results it is seen that there are eigenmodes of predominantly longitudinal vibrations as well as there are eigenmodes of predominantly transverse vibrations. Substantially different behaviors of the lower and upper layers of beam type and the internal layer of elastic body type are observed from the presented graphical results of the first eigenmodes of the investigated structure.

The presented results are applicable in the process of design of elements of agricultural machines and devices.

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Author for contacts:
Arvydas Pauliukas
Vytautas Magnus University
Studentų Str. 11, LT-53361, Akademija, Kaunas District, Lithuania
E-mail: arvydas.pauliukas@vdu.lt