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Didactic Principles of Visualization of Mathematical Concepts in Primary Education

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Abstract. Illustrations are traditionally used in primary education to explain mathematical reasoning, however, not every visual representation facilitates understanding of mathematical ideas. Therefore, the search for effective forms of visual representation of concepts is a topical issue of didactics. The purpose of the research, the results of which are presented in this article, is the development, substantiation of the didactic principles of concept visualization through the use of educational modeling and their implementation in the educational materials in Mathematics for primary school.

Keywords: *primary mathematical education, visualization, educational modeling, visual educational model, didactic principles of visualization.*

Introduction

The problem of visualization in mathematics teaching methods is traditionally associated with the primary stage of mathematical education, which is based on the specificity of mental activity of children in this age group. However, according to many psychologists and mathematicians, visualization is an important mental process not only for a child's thinking, but also for the mathematical style of thinking of an adult. In particular, A. Einstein answering the questions of the well-known questionnaire noted that the mental elements of his thinking are not words, rather signs or images (Adamar, 1970).

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In the modern educational process information visualization is understood in a broad sense as the use of visual representations of abstract data and knowledge to expand human cognition (Card, MacKinlay & Shneiderman, 1999). One of the first scientists to raise the issue of the use of visual representations in mathematical education was Littlewood (1953), however, the idea of visual thinking in mathematics and visualization in mathematical education became widespread only at the end of XX – beginning of the XXI century (e.g., Dalinger, 2006; Giaquinto, 2007; Hitt, 2002; Lakoff & Nunez, 2000; Mancosu, Jorgensen, & Pedersen, 2005; Reznik, 2012).

Visualization in the mathematical training of 6–10-year-old children is usually considered in conjunction with the idea of visual teaching – the general didactic principle of learning any substantive content. With regard to primary mathematical education, the idea of visualization of educational information was provided in the methodological recommendations of the renowned European pedagogues of the XVIII–XIX cc. A. Diesterweg, A. V. Grube, W. A. Lay, J. H. Pestalozzi, etc. However, so far the methodical science has not developed a common conceptual understanding of the ways of effective implementation of the idea of visualization of educational mathematical material in the practical development of modern textbooks for primary schools. As a result, cooperation of authors of textbooks with graphic designers does not always lead to a result planned by the authors.

The development of common principles of visualization in the primary mathematics teaching is based on the understanding of the pedagogical problems that can be solved with its help: for example, creation of motivation to study a topic, formation of emotional attitude, understanding of mathematical concepts being studied, remembering the necessary information, and others. We believe that the problem of understanding of the educational material by younger pupils should be seen as a priority task; and visualization is an effective means to solve this problem.

Problem Statement

The complexity of formulating the general principles of visualization of the educational information for the primary course of Mathematics, in our opinion, is caused by the fact that modern psychology has not found convincing answers to the important question of the methodological research about the essence of the process of understanding the educational material. A. A. Stolyar (1986) in his fundamental work “Pedagogy of Mathematics” writes that the science in this area is still at the intuitive level because the mechanism of understanding is still poorly understood. “The conclusion that a pupil understands (rather than merely knows) the material is only probable, but not credible” (Stoljar, 1986, p. 67). And though the book was published more than thirty years ago, the problem expressed by A. A. Stolyar is still relevant.

Visualization is a way to provide information to facilitate the understanding of mathematical ideas. However, in recent years, the scientific and methodological research on visualization in teaching mathematics has often been carried out by the example of teaching pupils of the senior age group (e.g., Giaquinto, 2007; Mancosu, Jorgensen & Pedersen, 2005; Reznik, 2012). Visual representations of mathematical concepts as a methodical idea *in training "adults"* was new in the didactic discourse and led to a keen interest in this issue and, in our opinion, was one of the main factors that caused publication of fundamental works in this field. In *primary mathematical education* visualization has traditionally been considered an obvious basis for learning, so scientific understanding of the problem of visualization in primary school has not appeared to be an urgent task in pedagogy. This led to the fact that in the last two or three decades the number of theoretical developments in this area has reduced, and the works themselves have mainly focused on individual methodological examples (e.g., a study of Bartolini Bussi and Mariotti (2008) on semiotic mediation by the example of the use of an abacus for teaching mathematics). As a result, we are now faced with a discrepancy between the established practice of the active use of visual representations in teaching mathematics in primary school and the lack of the proper theoretical grounding. Visualization is usually applied *ad hoc*, i.e. unsystematically, sporadically, and is not always aimed at solving the problem of *understanding* the mathematical material. Usually a solution of the problem of *understanding* in the actual practice of primary education is replaced by the problem of *motivating* pupils and forming *interest* in the subject with the help of emotive visualization.

The results of the long-term research conducted in schools of the Republic of Belarus have led us to the conclusion that the primary means of visualization of educational material in the primary course of Mathematics should be educational modeling that focuses on the essential aspects of the mathematical concepts being studied, which are often hidden from direct observation. Visualization of the key aspects of the mathematical theory is, according to many researchers, the main factor that influences the conceptual understanding of pupils (e.g., Bartolini Bussi & Mariotti, 2008; Davydov, 1996; Galperin, 1985; Salmina, 2012; Semadeni, 1984; Vertgejmer, 1987). Hence the problem of developing the principles of visualization of mathematical concepts by means of educational modeling that can be the basis for developing textbooks for primary schools becomes an independent and relevant pedagogical task.

The aim of the study is to develop, substantiate the didactic principles of creating visual representations of mathematical concepts by means of educational modeling and implement them in educational materials for primary school.

In accordance with the aim of the study, the following tasks have been solved consequently:

– to clarify the nature of concept visualization in mathematical education and to substantiate the viability of educational modeling as a didactic basis for the creation of visual representations of the concepts studied in primary school;

– to develop a classification of study models in primary mathematical education and to determine the role and place of visual models at various stages of learning mathematical concepts;

– to substantiate the didactic value of dynamic visualization and experimentally verify the effectiveness of traditional and computer dynamic visualization in primary mathematical education;

– to develop and experimentally test the method of forming the ability to construct and apply a visual model for solving mathematical tasks by pupils of grades 1–4;

– to implement the results of the study in educational materials in Mathematics for primary school.

The methodological basis of the study

Psychological and pedagogical theories about the role of the visual components of thinking in personality development and modeling as a special form of sign-symbolic activity in the school education: the specifics of human mental activity related to visual perception (R. Arnheim, P. Y. Galperin, R. Gregory, W. James, V. A. Krutetsky, etc.); the theory of development of the symbolic function of consciousness (J. Piaget, L. S. Vygotsky, etc.); the theory of educational modeling (V. V. Davydov, N. G. Salmina, etc.), the theory of the gradual formation of mental actions (P. Y. Galperin, N. F. Talyzina, etc..), didactics of primary mathematics (N. B. Istomina, A. A. Stolyar, G. Verne, etc.).

Study methods

Theoretical analysis of the philosophical, pedagogical and psychological literature on the topic under study; pedagogical experiment on the basis of primary school in the Republic of Belarus.

The first task of the study was to clarify the essence of concept visualization in mathematical education and to substantiate the viability of the use of educational modeling as a didactic basis for the creation of visual representations of the concepts studied in primary school.

The analysis of the literature on visualization in teaching mathematics shows that schematic images have a positive influence on formation of the concepts, while realistic images often distract from the essential aspects of the studied material. This is particularly evident in the work of teachers with pupils who are less able to abstract individual features in a particular object shown as an illustration.

Images rich in realistic detail do not provide mathematical ways to analyze the object and do not reveal its essential aspects. Schematic images, or *study models* of the concepts, contain the methods of action, which are easier for a child to transfer to other situations. Therefore, from our point of view, study models are a proper basis of visual representations of the mathematical concepts in primary school.

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Modeling provides for activity of the subject. A model can be considered as visualization of a mathematical concept only for a subject able to adequately perceive it, i.e. correctly associate the image with the formal and verbal representations of the concept.

As a result of the work carried out to solve the first task of the study, we have formulated *the principle of functional differentiation of model and non-model visualization of educational mathematical material.*

In accordance with this principle, textbooks for primary mathematics should organically combine model and non-model visualization with regard to their functional differences.

Model visualization means *visual representation* of the material with the help of study models that capture the essential aspects of the mathematical concepts being studied. The functions of the study models determine the main functions of model visualization: replacing the object being studied; generation of new knowledge in the process of operating the models; synthesis and specification of the mathematical material being studied, etc. The example of model visualization from the Mathematics textbook (Murav'eva & Urban, 2015) in Figure 1 shows each visual object (both object picture and scheme) reflecting the essential characteristics of the text of the task: the numerical data and the relationship between them.

There are 3 dolls on the shelf, and the cars are 2 more. How many cars are there on the shelf?
Dolls – 3.
Cars – ?, 2 more.

Fig. 1. The example of model visualization

Non-model visualization means *visual support* of educational mathematical material with images that are not study models and perform the following functions: to illustrate non-mathematical concepts and phenomena mentioned in training tasks; to realize interdisciplinary connections; to broaden the horizons and improve humanitarian orientation of mathematical training of pupils; to motivate pupils to learn mathematics by visually attractive means accompanying the text of training tasks, etc.

The task from the Mathematics textbook for grade 4 (Murav'eva & Urban, 2014) shown in Figure 2 is an example of non-model visualization.

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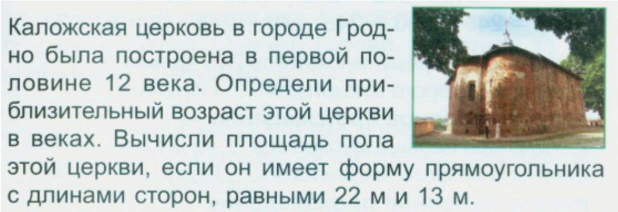

Kalozha church in Grodno city was built in the first half of the 12th century. How many centuries old is the church? Calculate the floor area of the church: it has rectangular shape with the sides 22m and 13m long.

Fig. 2. The example of non-model visualization

The second task of the study was aimed at developing the classification of study models in primary mathematical education and substantiation of the role and place of visual models at various stages of learning a mathematical concept. This task was completed with the formulation of the two principles: *the principle of priority of visual educational modules at the stage of familiarization with new educational mathematical material* and *the principle of multi-model transfer at the stage of consolidation of educational mathematical material*.

According to *the principle of priority of visual educational modules at the stage of familiarization with new educational mathematical material* visual study models in the process of learning a new topic must precede verbal and symbolic models. The implementation of this principle requires clarification of our approach to the classification of study models. From the point of view of the use of representation means in primary mathematics, all study models can be divided into three groups: visual models, verbal models, symbolic models.

Visual models include *object* models built with the help of real objects or their images, as well as *schematic* models based on the idea of substitution of real objects with geometric shapes. Object models are visually similar to the reality described in the training task, while schematic models do not have illustration similarity and allow abstracting from the unessential features of the objects being studied.

The illustration shown in Figure 3 is an example of an object model of a verbal arithmetic task from the Mathematics textbook (Murav'eva & Urban, 2014).

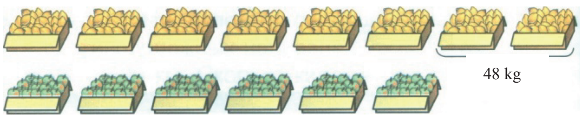

Fig. 3. Example of an object model

Schematic models are represented in two types – schematic illustrations and drawings. Examples of these schemes from the Mathematics textbook (Murav'eva & Urban, 2015) are shown in Figure 4.

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a *b*

Fig. 4. Examples of a schematic illustration (a) and schematic drawing (b)

Verbal models present the essential features of the object being studied in the natural language. Examples of verbal models include the text of the task and its brief record in a tabular or non-tabular form. An example of a verbal model in a tabular form from the Mathematics textbook (Murav'eva & Urban, 2014) is shown in Figure 5.

Скорость, км/ч	Время, ч	Расстояние, км
одинаковая	? } 8	15
	?	25

Fig. 5. Example of a verbal study model

Symbolic models represent the object being studied in the language of mathematical symbols. These models will include mathematical expressions, equalities, inequalities, and equations – all kinds of entries made in a formal mathematical language. Examples of symbolic study models corresponding to the offered text from the Mathematics textbook (Murav'eva & Urban, 2016) are shown in Figure 6.

Read the task. What do the entries mean?
Ivan was building a tower with a construction kit. He used 20 yellow, 10 red, and 5 blue pieces.

$\boxed{20 + 10}$ $\boxed{20 - 10}$ $\boxed{20 + 10 + 5}$

Fig. 6. Examples of symbolic study models

The priority of visual study models at the stage of familiarization with new material means that training mathematical tasks at this stage require the cognitive "route" from a visual model (object or schematic) through a verbal model to a symbolic model. The choice of visual models is determined, first, by the leading type of thinking of children of this age group (visual-active and visual-image thinking), and second, by the special role of the visual components in the mechanism of productive thinking, the development of which requires active stimulation from the first years of school education.

The principle of multi-model transfer at the stage of consolidation of educational mathematical material involves the use of different kinds of models to train transferring information from one "language" to the other. Such transfer at the stage of consolidation of educational material marks the transition to the actual modeling, which does not mean operating a single model, but the ability to create a new model based on the given one with the use of another code. To introduce pupils to modeling as a separate kind of activity, a special set of tasks is required that will equally represent different types of study models. Since the group of visual models includes object and schematic study models, four model categories were used for the compilation of training tasks: object, schematic, verbal and symbolic study models. Figure 7 shows all 6 pairs of models, which can be used to make 12 kinds of tasks to form the ability to relate one type of model to the model of another type. Only one pair does not include a visual model.

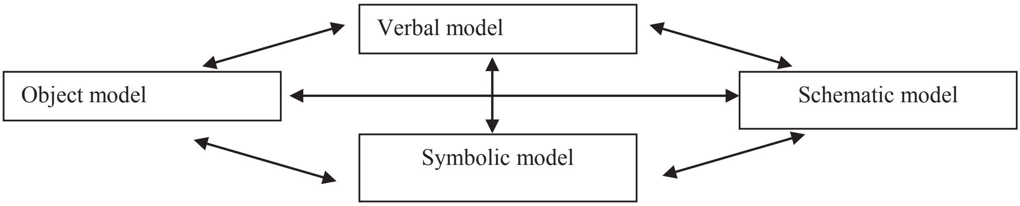

Fig. 7. Pairs of the models to compile training tasks in accordance with the third principle

The fourth task of the study was to substantiate the didactic value of dynamic visualization and to verify the efficacy of traditional and computer dynamic visualization in primary mathematical education by experiment. As a result of this work, *the principle of combining static and dynamic visualization for teaching* has been formulated. The key idea underlying this principle is the provision that 6–10-year-old pupils need not only visualization of the mathematical concepts being studied, but also active manipulation with "materialized" mathematical objects. Therefore, the effectiveness of visualization in teaching younger pupils is associated with the creation of the possibility of manipulating visual objects applying not only *static*, but also *dynamic* visual study models.

Effective means of dynamic visualization are boards, chalk, markers, handouts for pupils traditionally applied in primary education. Taking into account the need of manipulative activity for the formation of the concepts, our teacher resource books, which are a part of the developed educational materials, include the section of oral and practical exercises. Here are some examples of exercises to construct visual models from the teacher resource books for 2 grade (Murav'eva, Urban & Gadzaova, 2012) and 4 grade (Murav'eva, Urban & Gadzaova, 2014).

– Perform practical work: using 16 sticks make 1 square, 2 squares, 3 squares, 4 squares, 5 squares (Figure 8).

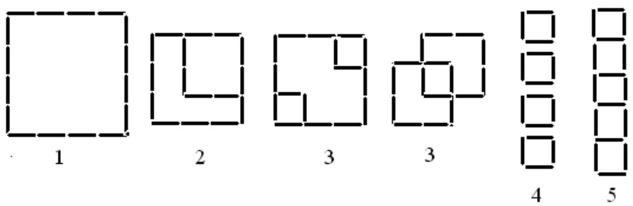

Fig. 8. Example of a task to construct a square model

– Pupils are offered to check whether it is possible to draw a shape with a single stroke (without lifting the pencil from the paper or drawing a line twice) (Figure 9).

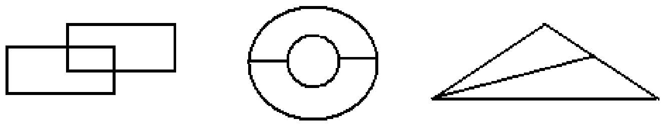

Fig. 9. Example of a task of the topological nature

– Perform practical work with a rectangle model: measurements are taken to calculate the perimeter. The task is to change the shape with the help of scissors so that its shape remains unchanged, and the perimeter is 2 cm less.
– Work with the *Tangram* manual. The task is to make shapes of birds with all the parts of the set (Figure 10) using the undivided template (without isolation of the components).

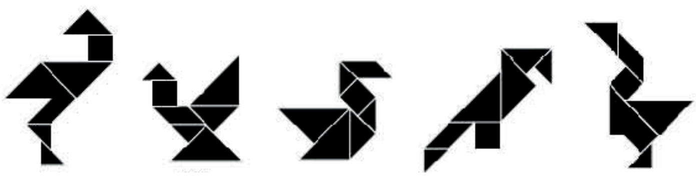

Fig. 10. Example of a task to construct shapes from the Tangram manual

Modern computer technology allows designing high-tech teaching tools to solve the problem of dynamic visualization on a fundamentally different level of quality. An example of realization of the idea of computer dynamic visualization of mathematical concepts in primary mathematical education is our electronic teaching aid (ETA) "Mathematics. Grades 2–4" (Èlektronnoe sredstvo obučenija, 2010), which is a component of the educational materials in Mathematics for primary school. In the course of development of the ETA we took into account the experience of the implementation and realization of two benchmark projects in the field of computerization of education: the well-known computing environment LOGO (Papert, 1993) and The Physics Education Technology Project (PhET) (Wieman & Perkins, 2005).

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The advantage of computer dynamic visualization over static one is that it makes it possible to consider the dynamics of the genesis of a new mathematical object (Sergeev & Urban, 2012). ETA "Mathematics. Grades 2–4" allows pupils to manipulate the images that are the models of the concepts being studied. Many interactive models of the ETA are based on the well-known examples of cultural artifacts (abacus, clock, scales, chart, etc.) that are still used in school teaching. The computer analogue in comparison with a real artifact allows constructing non-trivial training tasks and has an overall impact on changing the methods of work with visual models.

In the course of the study experimental comparison of didactic effectiveness of static and dynamic computer visualization was performed. The comparison was made in the context of work with interactive models that visualize the dynamics of work with an abacus, clock and scales. We will demonstrate the results of the study by the example of work with the interactive model "Clock". Work with the interactive model "Clock" has a number of significant differences when compared with the traditional dynamic visual media (paper model of a clock with moving hands). Here are some of them:

– it creates conditions for each pupil to work with the clock model at their own pace, realizing their own research hypotheses;

– it provides an instant connection between the visual representation of the time on the clock dial and the abstract representation of the named number;

– in case of an erroneous execution of the task the dial will show the second pair of the hands corresponding to the recorded reading of the clock (Figure 11 *a*).

– it is possible to work simultaneously with two clock dials, solving the tasks to determine the duration of an event on the basis of the simultaneous visual perception of the beginning and end of the event (Figure 11 *b*).

– availability of the second pair of the hands on the clock dial allows more effective solving of the tasks to clarify the concepts of "quarter of an hour", "half an hour", "less than half an hour", "more than a quarter of an hour", etc. It promotes the formation of the ability to perform approximate calculations with the units of time in life situations that do not require the accuracy of calculations (for example, to answer the questions "Will I catch the train?", "Do I have enough time to…", etc.) (Figure 11 *c*)

a b c

Fig. 11. Work with the interactive computer model "Clock": a – visualization of a mistake; b – work with two clock dials, c – visualization of the concept "quarter of an hour"

After a series of lessons with 4th grade pupils from Borovliany we hold the final control tests in the experimental class, where ETA "Mathematics. Grades 2–4" was applied (24 pupils), and in the control class, where the teacher applied traditional visual means of teaching the topic (25 pupils). The content of the first part of the control tests included typical questions to determine the time on the clock. This part of the tests was done with about the same success by the pupils of the two classes, which suggests practically equal methodical efficiency of teaching with or without the ETA for the typical situations of measuring the time, where pupils come to a result by performing arithmetic operations of addition and subtraction.

However, the final control tests included the second part with unconventional tasks. It is important to note that the pupils had not dealt with these tasks during the training. In these tasks we asked the pupils to solve practical problems not associated with the need to accurately determine the clock reading or calculate the duration of the event within the accuracy of a minute. On the contrary, all tasks in this section were designed to test the ability to estimate the clock reading approximately for quick orientation in time. For example, a picture showed two clock dials. Each of them had only the minute hand. The pupils were asked to determine whether it could be an hour and a half between the beginning of the event (the first dial) and its end (the second dial).

75% of the pupils from the experimental class, where the training had included the use of interactive computer models, dealt successfully with such tasks, as compared to only 32% of the pupils from the control class, where traditional dynamic models had been used. The experimental training conducted with the help of ETA "Mathematics. Grades 2–4" led to the conclusion that, if the methodological goal is to teach the ability to accurately determine the time in standard situations requiring arithmetic operations with numerical values of time, electronic teaching aids do not provide didactic advantages over traditional teaching aids. However, if the methodological goal is to teach the ability to orientate in time in unusual situations, where the search for the answer does not require performing arithmetic operations, electronic teaching aids demonstrate a significant didactic advantage as compared to traditional teaching aids. This experiment was described in detail by Urban (2014).

In the course of the work on the fifth task of the study, we had to develop and experimentally test the methodology of forming the ability to construct and apply visual models to address educational mathematical tasks. The work in this direction allowed us to formulate *the principle of gradual formation of the ability to operate a visual model*. According to this principle, the ability to construct and apply visual models can not be formed at the proper level of generalization required to continue education at secondary school without providing a special teaching environment including the sets of didactic tasks. Our experiments conducted in the final grades of primary school showed that if the ability to model had not been a special teaching goal, the pupils usually preferred to construct object study models, rather than schematic ones, or did not try to visualize the search for solutions to the tasks at all. As an example, here are the results of one of the experimental tests we conducted in 2012–2013, where pupils from fourth grades (Borovliany Secondary School, Zaslavl Grammar School) were asked to solve some non-standard tasks. We asked the children to record their calculations and drawings on separate sheets of paper in case of difficulties. Of the 89 pupils, only 9 (10%) constructed a schematic model to the task, 20 pupils (22%) made object drawings, and the rest of the pupils (68%) did not take any attempts to find the solution to the non-standard task other than repeated reading its conditions aloud or to themselves. In our opinion, this was not caused by the inability of younger pupils to construct visual models, but was due to the fact that the children had not been taught to do that specifically.

Purposeful formation of the ability to operate a visual model requires compliance with the following stages of training, specified by us in the course of the study:

– Learning to *relate* study models. Solving tasks at this stage, a child must determine whether the models offered for comparison correspond to each other and explain why they correspond (or do not correspond).

– Learning to *select* a visual model. Solving tasks at this stage, children have to select from multiple visual models the one that corresponds to the offered verbal or symbolic model.

– Learning to *complement* a visual model. Solving tasks at this stage, pupils must introduce the necessary additions to the offered visual model so that it is consistent with the verbal or symbolic model.

– Learning to *construct* a visual model. Tasks of this stage are aimed at developing the ability to construct their own visual model corresponding to the verbal or symbolic model.

– Learning to *transform* a visual model. The tasks of this group require from pupils to make changes to the offered or their own constructed visual model to find new ways of solving the task.

The developed methodology of teaching the construction and application of visual models for solving educational mathematical tasks was implemented in the authors' textbooks for grades 1–4 and evaluated in the course of experimental work with pupils of third grades of Borovliany Secondary School, Zaslavl Grammar School, and Minsk Grammar School No. 10 in 2013–2014. 69 pupils from the experimental classes and 73 pupils from the control classes took part in the training. As a result of application of the developed methodology, 86% of the pupils from the experimental classes have formed the ability to construct visual models to the texts of tasks, 43% of the pupils have formed the ability to transform a constructed model. In the control classes of these schools, where the authors' textbooks had not been applied, 52% of the pupils demonstrated the ability to construct visual models; the ability to transform a constructed model was demonstrated only by some pupils (12%).

The sixth task of the study was aimed at the implementation of the study results in an educational complex (educational materials) in Mathematics for primary schools of the Republic of Belarus. The results of the work in this direction are the educational materials in Mathematics for grades 1–4 of schools of the Republic of Belarus, which include: Mathematics textbook, workbook, test notebook, notebook for stimulating work, electronic teaching aid for pupils of grades 1–4, and teacher resource book published in the Belarusian and Russian languages in the period from 2011 to 2014.

Conclusion

The conducted study, which results are presented in the article, leads to the following conclusions:

– The didactic basis for creating visual representations of mathematical concepts in primary school is educational modeling that allows capturing in the image a number of significant, yet often not obvious aspects of the concepts being studied. This is a key factor influencing the conceptual understanding of the mathematical theory by younger pupils. This provision was generalized in *the principle of functional differentiation of model and non-model visualization of educational mathematical material*.

– For the classification of study models according to the method of representation, it is useful to identify three main groups: visual, verbal and symbolic study models. At the stage of familiarization with a new mathematical concept in primary school, the priority role belongs to visual study models, and at the stage of generalization and systematization of educational mathematical material, visual study models are not of the greatest significance. The didactic learning effect at the stage of generalization of the knowledge is provided through the practice of the balanced application of the different kinds of models and transfer of information from one code to another. Summarizing the results of the work in this area led to the following two principles: *the principle of priority of visual educational modules at the stage of familiarization with new educational mathematical material* and *the principle of multi-model transfer at the stage of consolidation of educational mathematical material.*

– In the process of teaching pupils of primary school, static visual models must be applied in combination with dynamic ones that provide a valuable kinesthetic experience for pupils and an opportunity to actively manipulate the created images. The modern technological context offers additional possibilities for the application of dynamic visualization in educational practice. The comparison of the didactic effectiveness of the traditional and computer means of dynamic visualization has proved that the resources of computer visualization provide a higher level of understanding of the material in non-standard educational situations. Application of the traditional and computer dynamic visualization showed practically no differences in the level of educational achievements of the pupils when solving typical educational tasks. The results of the work in this area have been summarized in *the principle of combining static and dynamic visualization in the teaching process*.

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– Teaching to construct visual models requires the creation of a special educational environment and the application of the methodology of the gradual formation of the ability to model. Without the application of the special methodology only individual pupils developed the ability to construct visual models, yet not comprehensively. This provision was the basis for the formulation of *the principle of gradual formation of the ability to operate a visual model*.

– The didactic principles of visualization of educational material with modeling tools formulated in the course of the study formed the basis of the authors' educational materials for Mathematics for school grades 1–4 in the Republic of Belarus.

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Matematikos sąvokų vizualizacijos pradiniame ugdyme didaktiniai principai

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Santrauka

Matematinių sąvokų vizualizacijos idėją tiria psichologijos ir pedagogikos mokslas nagrinėdamas įvairius matematinio ugdymo etapus. Vizualizacija yra ypač efektyvi pradiniame matematikos ugdymo etape, nes ji atitinka 6–10 metų amžiaus kognityvių procesų ypatumus. Todėl pradinio ugdymo vadovėliuose dažnai pasitelkiamos iliustracijos, siekiant paremti matematinius argumentus. Tačiau pastarieji teoriniai tyrimai ir mokyklinio ugdymo praktika liudija, kad ne kiekvienas vizualus pateikimas padeda geriau suvokti matematines idėjas. Todėl, norint tinkamai įvykdyti didaktinius tikslus, šiuolaikiniai matematikos vadovėliai, skirti pradiniam ugdymui, turi pateikti tinkamą matematinių idėjų vizualizaciją.

Šiame straipsnyje pateikiami penki didaktiniai principai, kuriais grindžiamas matematikos idėjų vizualinis pateikimas: 1) modeliu grindžiamos ir nemodeliu grindžiamos edukacinės matematinės medžiagos vizualizacijos funkcinė diferenciacija; 2) prioritetas teikiamas vizualiems

ugdymo moduliams supažindinant su nauja edukacine matematine medžiaga; 3) kelių modelių perkėlimas edukacinės matematinės medžiagos konsolidavimo etape; 4) statinės ir dinaminės vizualizacijos derinimas mokymo procese; 5) laipsniškas gebėjimo valdyti vizualaus modelio formavimas. Šie principai tapo pagrindu naujiems Baltarusijos pradinio matematinio ugdymo vadovėliams.

Esminiai žodžiai: *pradinis matematinis ugdymas, vizualizacija, edukacinis modeliavimas, vizualinis edukacinis modelis, didaktiniai vizualizacijos principai.*

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