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Knowledge of Mathematics Teachers of How to Teach 8th Graders Content of Triangles

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Abstract. Study aimed to document the 8th grade math teachers' knowledge of how to teach the triangles topic, Hashweh's Model (2005) was accredited for this purpose. The model included the following components: knowledge of the pedagogical content, beliefs about learning and teaching, learners' characteristics, educational strategies, educational context, learning sources, curriculum, and clarity of the content and philosophy. The (female) teacher whose students obtained the highest degrees in TIMSS (2015), and the (female) teacher whose students obtained the lowest grades in this test were selected for this study. The researcher applied the qualitative method to collect the required data using different tools, such as questionnaire, oral interviews, classroom observations, content test, and document analysis. The results showed that the teachers' knowledge of how to teach the content varies. The teacher whose students obtained the highest degrees in TIMSS (2015) test did better in all the knowledge aspects: knowledge of the pedagogical content, beliefs about learning and teaching, learners' characteristics, educational strategies, educational context, learning resources, and curriculum. On the other hand, their knowledge was equal about the educational objectives.

Keywords: *pedagogical knowledge, constructivism, mathematics performance, teacher classroom practice, TIMSS.*

Introduction

Teacher's knowledge has been a subject matter of intensive research for a long time (Ball, 2008; Bray, 2011; Petrou & Goulding, 2011). The scope of knowledge the teachers

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rely upon every day is wide: content knowledge, students, curriculums, pedagogy, and psychology. However, studying one aspect of the teacher's knowledge independently is not only unrealistic, but also difficult. Even though, to study the teachers' knowledge of math teaching, it is unavoidable to study how to teach the pedagogical content knowledge (PCK) (Schmidt, Cogan, & Houangm, 2011), to satisfy this purpose. Shulman (1986) applied PCK for the first time when he tackled the missing pattern in education, namely, knowledge of subject matter. PCK, as dealt in by Shulman, included two basic elements: knowledge of representing certain subject knowledge to make it understandable to others; and knowledge of the learners' characteristics, alternative concepts, and learning disabilities they face during learning (Shulman, 1987).

In the beginning, Shulman considered PCK a part of content knowledge, in addition to curriculum knowledge and knowledge of subject matter. Then, in 1987, retackled it and classified it into seven types of the teacher's professional knowledge, which included: content knowledge, general teaching methods, curriculum, content teaching methods, learners' characteristics, teaching context (learning environments), and objectives and philosophy.

Later, Grossman (1990) added two other elements to PCK concept: knowledge of beliefs and objectives, and knowledge of the curriculum subjects. Several researchers were interested in studying how to teach the content. Many studies agreed on defining PCK as integration between how to teach and content understanding, so that it will produce best learning with the students to understand the content.

The purpose of this study is to see PCK in the classroom, the talk of the teacher about his/her experience; and to see a blend of overlapping elements, which, when united, help in providing an internal vision of this concept (Loughran, Mulhall, & Berry, 2004). Hashweh (2005) developed a model of how to teach the pedagogical content knowledge (PCK), which consists of many components, as per the following details.

Knowledge of the pedagogical content: the teacher's knowledge of the topic he/she teaches should be deep, not limited to the prescribed material in the schoolbook. He/she should also master all the concepts and scientific terms and relations between them, their teaching methods, and how to interrelate these concepts and knowledge in a manner that makes it more convenient to the students, and avail them a chance to better understand it.

Beliefs about teaching-learning and learners' characteristics: this includes knowing the students' trends, interests and abilities, their individual differences, the teaching disabilities that may face them during learning a certain subject. It also includes their previous knowledge, the misconceptions they may hold, means of handling and getting rid of them, and the suitable teaching strategies for them.

Knowledge of the teaching strategies: this knowledge requires the teacher to be aware of the representations and simulations, and the language he/she uses to explain the educational content to his/her students, which will render the subject more understandable for them. In addition, he/she should know the suitable educational strategies for the ed-

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ucational content, such as use of the lab, activities, lecture, cooperative learning, drama and other teaching strategies. It is essential that he/she should be aware of the methods that reveal his/her students' misconceptions, and how to handle them. The knowledge further includes the strategies the teachers will follow when they evaluate their students, such as examinations, homework, addition assignments, and type of questions they will raise to their students, i.e. will they measure lower or higher mental skills? The teacher should be alert about the significance and types of the questions he/she asks, and should build the educational strategies that will help the student build his/her knowledge by his/her own, while he/she remains the axle of the educational process.

Knowledge of the context: here the teacher should be aware of the environment that occurs during the learning process, whether a classroom or local environment. He/she should possess ability to make good use of both environments resources in a good way, and employ them to serve the teaching process.

Knowledge of learning resources: teacher's knowledge of the local and external learning resources, which enhance his/her students' learning, such as use of internet, worksheets, books, and the like.

Knowledge of the curriculum: this includes teacher's knowledge of the general curricular objectives, sequence of the topics in the curriculum and method of presenting during the academic years, so that he/she will introduce them in a manner that fits his/her students. Furthermore, he/she should know how to develop the concepts and know-

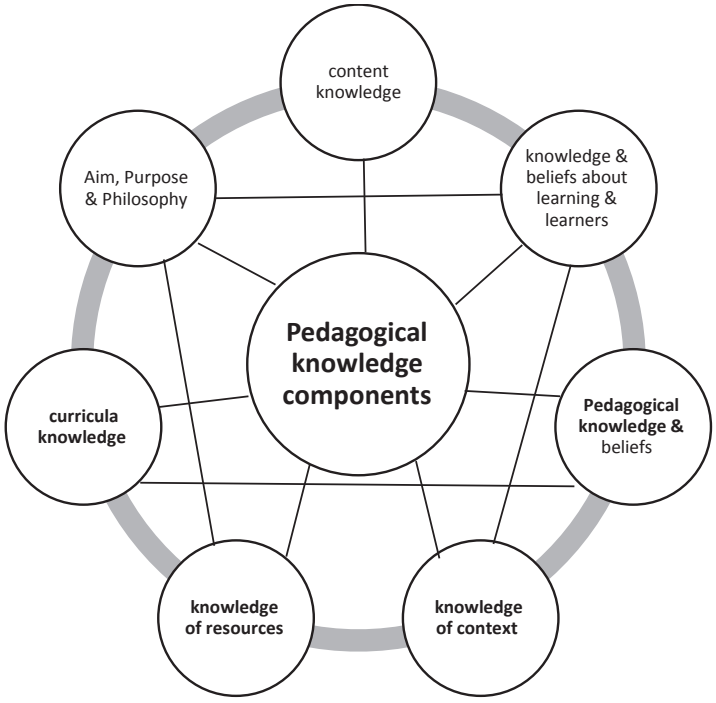

Fig. 1. Pedagogical knowledge components

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ledge through the curriculum, and ability to build the curriculum and reintroduce it to his/her students in a better way, which fits their cognitive abilities and characteristics

Finally, clarity of objectives and philosophy: the teacher should be quite aware of the general and special educational objectives he/she aims to achieve through teaching a certain topic, and explain these objectives to the learners. In addition he/she should be familiar of the knowledge and skills the student will eventually acquire, as well as knowledge of the factors that affect realization of these objectives.

The idea of the pedagogical content knowledge is built on the constructive view, which stemmed from focus on the thinking taking place inside the learner's mind, planning and decision-taking processes, being an active part of the teaching process, given that he/she is able to build his/her knowledge by his/her own (König, Blomeke, Paine, Schmidt, & Hsieh, 2011), through the accommodation and assimilation processes (Fig. 1).

Constructivism sees that the teacher's function is to facilitate and direct the teaching process by generating an active and effective classroom environment, which takes into account the learners' characteristics; teaching context; content; educational objectives and strategies; curriculum; and educational resources (Brosman, 2014).

Many research works were made about different aspects of the teacher's pedagogical content knowledge. Some of these works were about the effect of the teacher's knowledge of the material on his/her teaching (Wilkins, 2008; Hill & Lubienski, 2007; Ball, Thame, & Phleps, 2008). Other works focused on the former knowledge and its effect on building the new concepts (Love & Kruger, 2005). In addition, research works were conducted on problem-solving methods and others on the effect of the teachers' cognitive beliefs on their knowledge how to teach the content (König et al., 2011).

The current study depends on Hashweh's Model (2005) to identify the knowledge of the math (female) teachers of how to teach the triangles topic for the 8th graders, in terms of: teacher's knowledge of the topic content, objectives, teaching methods, aids, curriculum, resources and context. Where the researcher found, through the follow-up to the results of (TIMSS, 2015) and there are significant differences in the results of students in Jordanian schools, prompting the researcher to identify the knowledge of the math (female) teachers of how to teach the triangles topic for the 8th graders, And how it affects student results in the TIMSS test.

This study is important because of the importance of the role played by the teacher in the educational process, where he is taught by a large number of students, which increases the sensitivity of this role and dealing with caution during the teaching, to work on the development of concepts and students to give them properly. This is in the ability of the teacher to identify the characteristics of his request and the diagnosis of their knowledge in order to employ them in the educational process and correct the fault, if any.

The researcher hopes that the results of this study will lead to a qualitative shift in the pre-service and in-service teacher training programs by focusing on the knowledge of how to teach the content. It helps to provide an example that shows how to teach the

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teacher for a specific content, especially since recent years have witnessed the application of the new Jordanian curriculum requires the teacher to follow new methods and deal differently with the curriculum, which depends on the student.

Methodology

The researcher applied the qualitative, descriptive and analytic method to describe and document the math teachers' knowledge of how to teach the content of the triangles topic for the basic eighth graders, through the following tools:

Beliefs questionnaire about learning-teaching: it aimed to distinguish between the teachers who hold behavioral and constructive beliefs about learning-teaching. It included 32 items based on selecting from multiple choices, namely four alternatives: *strongly agree, agree, disagree and totally disagree*. The teacher has to select one only. The items explored eight dimensions of the social constructivism, each has four questions, and the maximum point is 4 and the lowest is one. Accordingly, the highest degree of the dimension is 16 and the lowest is 4. The questionnaire was constructed by Hashweh (1996). Thereby, the maximum degree the teacher may obtain is 128 points, and the lowest is 32. The questionnaire aimed at identifying the teachers who obtained 96 points and more, i.e. three quarters of the maximum degree. In this concern, if the teacher obtained pointes between (96 and 128), that means she holds constructive social beliefs; and if she obtains less than (96) points, that means she holds behavioral beliefs. The researcher verified the questionnaire validity and reliability, as its reliability coefficient was (0.81). Table 1 illustrates the questionnaire items.

Table 1
Beliefs Questionnaire about Learning-Teaching

	Constructive, Social View about Learning	Traditional View about Learning	No. of Items
A1	The learner is active, has motive to understand, and builds knowledge to achieve this.	The learner changes his behavior if it is positively enhanced.	4
A2	The learner develops many ideas by his own to use in assimilating new ideas.	The learner does not have many ideas about math before learning, and previous knowledge is important only as a prerequisite.	4
A3	Many of the previous ideas (misconceptions) contradict the scientific ideas.	There is no perception of the existence of alternative conceptions.	4
A4	Math learning is mostly a conceptual change process.	Math learning is a gradual process, represented in increasing or augmenting the learner's knowledge in an accumulative manner.	4

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	Constructive, Social View about Learning	Traditional View about Learning	No. of Items
A5	Optimal teaching methods require facing the misconceptions.	Optimal teaching methods require ignoring the alternative conceptions, if any.	4
A6	Social interaction with teachers and peers is essential to find a common meaning and obtain the required support or attribution.	No need for social interaction, as the individual work is the most important.	4
A7	There dialogue is important in teaching.	The dialogue is of no importance in teaching.	4
A8	The significant knowledge is interrelated concepts used in understanding and assimilation.	The significant knowledge is facts, rules and equations, learnt by heart and retrieved upon demand.	4

Classroom Observations: aimed at monitoring the teachers' practices, classroom discussions, and examinations, to provide direct information about the teacher's knowledge of the content, her beliefs, and how far she knows her students and their responses. It also provides information about the teaching methods, activities, and the evaluation methods the teachers use, taking into account the classroom environment where the observations were made, taking the number of the student in the class, depending on both audiovisual recording and taking observations. The researcher made 24 observations (12 per each participant).

For reliability, before starting the observations, the researcher and his colleague performed three observations of a math teacher in one of the study population schools, to have training on the classroom observations. Thereafter, the researcher and his colleague attended different classes of the teachers, to observe the teachers' performance in the different subjects they taught. Both, separately, wrote down the observations and analysis; the analysis process was continuous, subject to ongoing feedback, and was carried out as follows:

A- Downloading the audio records on papers and reading them more than once, to look for the PCK components that appeared in them, by setting definite aspects of each component of the content knowledge. Then, we searched for the existence (or nonexistence) of these components with the two (female) teachers in the classroom observations.

B- Carrying out the oral interviews to support the classroom observation, which were also downloaded manually, after hearing them more than once for both teachers, to look for the aspects of the content knowledge components. Thereafter, they were compared with the classroom observations.

C- The analysis reliability was measured by comparing the compatibility between both analyses (the researcher's and his colleague's), which was (83%).

1- Content test: an examination was constructed for the teachers to test their knowledge of the "triangles" content, which was made as per the specifications table. It contained

the main ideas provided in the unit, the previous knowledge they require, extent of the teacher's knowledge of the misconceptions, students' learning disabilities, and revealing their knowledge about the curriculum integration, both vertically and horizontally. The exam consisted of two sections. The first consisted of 26 multiple choice questions, so that each question has four alternatives. The second consisted of five essay questions. The exam was presented to five arbitrators, and the amendments were made according to their recommendations. The test reliability was verified by calculating Cronbach Alpha coefficient, which amounted (0.84).

2- Individual oral interview: two interviews were made containing open-end questions, which aimed at identifying the teacher's beliefs about the learning-teaching, learner's knowledge of the unit content, teaching methods, evaluation, students' characteristics, curriculum, other resources, and anything having effect on the teaching process, as viewed by the teacher. The first interview was before the teacher began teaching the unit, and an audio recording was made to download and analyze it later by rehearing more than once, to make sure of the learner's responses. The second oral interview was after the teacher ended teaching and evaluating the unit, also with audio recording. We checked the validity of the interview by presenting the interview questions to the specialists, and the questions were amended based on their directions.

3- Documents: to obtain more accurate results, we used the worksheets, achievement tests, and conceptual maps prepared by the two teachers, for providing maximum amounts of data, which reflect evidences on the teacher's knowledge.

Participants

The study population consisted of all the eighth graders math teachers in all the public schools in Zarqa Governorate (Jordan), who participated in the international study of science and math (TIMSS, 2015) (n = 13). The researcher selected two (female) teachers intentionally. The first, was whose (female) students obtained the highest points in TIMSS (2015), whose performance average was (459.13), as it is the closest to the medium achievement level, and included the students' level who obtained 481 points and more. The second (female) teacher, was whose (female) students had the lowest points, whose performance average was (335.72) points. Both teachers agreed to participate in the study.

Finding AndDiscussion

1-(A) **A brief about the teacher in the high achievement school in TIMSS (2015).**

The teacher is in her thirties, holds BA degree in mathematics from a Jordanian university, spent 15 years in teaching. Currently, she teaches math to the eighth and ninth graders, she loves her career and likes dealing with the female students. She sees that teaching is a high mission that should be performed perfectly, and feels high belonging to the school where she teaches.

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- Good constructive view about learning-teaching

The responses of the teacher on the beliefs questionnaire about learning-teaching were recorded, and she got 108/128. Accordingly, she is classified among the teachers who hold constructive beliefs; her responses to the questionnaire items ranged between 10 and 16. She obtained 16/16 on the importance of dialogue in teaching domain, and 10/16 in math learning process as, mostly, a concept change process. Therefore, the teacher is closer to the social constructivism as she places great importance to the social interaction role and dialogue role in education.

The constructive aspects were very clear in her classroom practices and during interviewing, when we noted that she possesses positive attitudes toward education. She believes in connecting the math with the students' daily life; she expressed that in the interview when she said: "*Mathematics is very important in our lives.*" She sees that connecting math with the students' lives generates positive trends with them. For instance, she connected Pythagoras theorem to the students' lives through the following example. *"Imagine with me that you went to a restaurant to eat pizza. You found in front of you a 12-inch diameter, medium size pizza, and a 16-inch diameter large size one. Which one will be larger for you? If we applied Pythagoras theory:* $16^2 = 12^2 + S^2$*. Then, (S) value will be 10.5, i.e. the large pizza can be divided into the medium size one in addition to another smaller pizza. In this case, of course, two medium size pizzas are larger than one large size pizza. With a look at the prices, you will know which one will be economically better for you."*

Other evidence on the teacher's constructivism is that she places great importance to the dialogue in teaching. She said: *"Dialogue and discussion are very important. Sometimes I ask my students questions and ask them to conclude the information."* She also uses it to detect the learning disabilities and misconceptions the students face. She sees that revealing these problems is a vital matter. She does her very best to treat these difficulties by explaining them in other ways, or using various representations such as: conceptual maps, simulations, But this aspect did not appear in the classroom observations, possibly because the students did not encounter any misconception in that stage. Moreover, it seems that the teacher believes in the importance of the social interaction among the students. She used the group method in teaching her class, taking into account the individual differences, motivated by the development of the cooperation spirit among them.

Through answering the questionnaire items, we noticed that the important knowledge for her is interrelated concepts used for comprehension. She related the information she taught in the seventh grade by asking oral questions. She did not show a clear constructive view about the role of the teacher-learner in teaching. This was clear when she was asked the question (What is the job of the teacher and learner, in your view?). Her answer was: *"The job of the teacher is to perform his/her mission perfectly, communicate the information- which are in the book- to the student, and relate them with reality. And the job of the learner is to take care of the material, follow with the teacher the required things and solve the assignments given to him/her."* She was closer to constructivism in her

classroom practices, as she was facilitator and director of the education process. She assists the students in the groups, leads the dialogue and discussion among them, distributes the required tools, directs oral question to the learners to build on their responses, and motivates them to think and conclude the information.

She also believes in the importance of enhancement in education. She said, *"A single word may create with them motivation and self-confidence."* However, in her classroom practice, she did not use enhancement except in few occasions, which was oral use of the same words *"well done"*. Here we can see that the beliefs the teacher holds are not necessarily required to be consistent with his/her classroom practices, or vice versa. She was constructive in her behavior inside the classroom, and in her teaching methods. Yet, she was not aware of the meaning of constructivism when we asked her about it in the interview. Nor had she a clear educational philosophy, as she practiced them based on her experience in teaching, not on her theoretical background.

In the matter of the triangles content, she sees that it is a very important cognitive material, applied in our life to a wide extent. She believes that the unit is suitable to the students once communicated to them properly, such as diversification in strategies and use of representations, which will make it easier and more understandable. By contrast, students may find it difficult if given to them through the prompting style, because the material is untouchable for them. This is emphasized by the studies of Blum and Hill, Blunk, Charalambous, Lewis. Phelps, Sleep and Ball, (2008); and Krauss, (2008), that the use of simulation helps the learners understand and detect their misconceptions. In addition, it plays an important role in rebuilding their conceptual framework.

From the above, we conclude that there are constructive beliefs with this teacher, most of which were in harmony with her classroom practices. For instance, she adopts dialogue and discussion in teaching, and social interaction among the peers, considers the individual differences, and relates the new knowledge with the previous knowledge, which was reflected on her students' achievement in TIMSS (2015). She focuses constructivism on the learner and her activity during the learning process. She stresses on the meaningful learning, which is built on understanding through the active role and students' effective participation in the activities they support, to build their concepts and scientific knowledge. These results are in line with the study of Baviskar, Hartle and Whitney (2009).

1(B)A brief about the teacher in the low achievement school in TIMSS (2015):

The teacher is in her forties, holds BA degree in mathematics from one of the Jordanian Universities, with 18-year experience in teaching. Currently, she teaches the eighth, ninth and tenth grades, loves her job and holds positive attitudes about math, and sees that there is a close relationship between math and our life.

- Contradicting constructive view about learning-teaching

The responses of Teacher (2) on the questionnaire of the beliefs about learning-teaching were evaluated, and she obtained (90/128) degrees. Accordingly, she could be classified among the teachers who hold behavioral beliefs. Here, we have to point out that the degree

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is not an accurate standard upon which teachers could be classified. In other words, the teacher may obtain low degree while his/her practices are constructive. Her responses on the questionnaire ranged between 7 and 14 points. She obtained (14/16) degrees in the domain concerning the importance of dialogue in teaching, and (7/16) in the domain concerning the math teaching process as a conceptual change process, most of the times. As such, we can judge that this teacher is close to the social constructive view, as she places great attention to the social interaction role, and the role of dialogue in teaching.

All the research instruments, i.e. questionnaire, oral beliefs, and classroom practices, showed the presence of contradiction between what Teacher (2) holds of philosophy and beliefs, and her classroom practices. We discovered that the teacher holds positive attitudes toward teaching through her vision that the purpose of teaching is clarifying information to the students in a proper manner. Yet, she could not achieve this objective during her classroom practices in teaching the unit, although she is aware of the math importance and its role in the students' lives. She said: *"Mathematics is the most important subject taught to the students, because it shares everything in our life; it is an inseparable science from our life."* However, she could not relate the triangles topic to the students' life.

On the other hand, practices of Teacher (2) were not consistent with her beliefs, as she was far from constructivism by employing dialogue in her teaching. She asked her students few questions only, with few listening too. When we asked her about the teaching methods she employs to explain the unit, she indicated that the only *"successful method is the lecture"*. The researcher discovered that limiting her self to re-explaining the information, while maintaining the lecture method, failed in handling the misconceptions the students fell in. The same mistakes were recurrent in more than one lesson. She was careful to aware her students not to fall in these errors, which could be ascribed to her long experience in education, teaching the same subject.

In addition, Teacher (2) believes in the importance of the social interaction among the learners, and observes the individual differences among them. These aspects were very clear in her responses on the questionnaire articles about the beliefs, but none of these aspects appeared in her practices that indicate applying such beliefs. This assures the existence of a gap between what she holds of beliefs and what takes place in the classroom.

Walshaw (2012) indicated this finding in his study providing that the teachers' concepts about science do not necessarily affect their classroom practices. The teacher sees that the important knowledge is interrelated concepts used for assimilation; but she did not relate the information the students learned in both the seventh and eighth grades with the unit she teaches. Teacher (2) has a clear constructive view of the role of the teacher and learner in education. When she was interviewed, she answered, *"The job of the teacher is to be director and guider of the class, and teach the student how to obtain the information through his/her personal diligence, such as research. We have to deal with the student as a source of information not a "storage machine or information recipient"."* In spite of this, her classroom practices did not show this perspective, as her role was

limited to explain the material using the lecture method, and directing oral questions to the students, which rely on remembering.

Beliefs and teaching practices of Teacher (2) need to be reconsidered, as she believes in the external enhancement more than the internal motivation for students' learning, which was reflected in rewarding the students by giving pieces of sweets when they gave correct answers. She also believes that the ideas the students possess are not more than previous knowledge learnt in previous grades, and she depends on activating this knowledge. She is aware of the misconceptions with the students, but she does not possess the sufficient knowledge on how to face them, and how to make the proper conceptual change to them.

The above details indicate contradictions between the constructive beliefs of Teacher (2) and her classroom practices. She was aware of the role of dialogue and social interaction among the peers in teaching, the individual differences among the students, and the importance of relating the new knowledge to the previous knowledge. Nonetheless, these aspects were not present in her classroom practices, which is in line with results of the study of Zazkis and Leikin (2010). Possibly, this explains the low marks her students obtained in TIMSS (2015), a case confirmed by Gordon (2009).

2 (A) Teacher (1): Disparity in knowledge between the general and special objectives.

The researcher found that Teacher (1) seeks to achieve many objectives, such as the general objectives and those unit-specific. She indicated that the objective of mathematics lies in interpreting the natural phenomena and linking math with the students' lives, and described *"math as the core of life"*. She explained to her students the importance of mathematics in life, which indicates the existence of genuine general objectives she is seeking to achieve.

As for the special objectives, Teacher (1) was aware of them to a wide extent, which was apparent in both the interview and the classroom observations. She achieves all the objectives stated in the schoolbook, and was clear in her performance in the classroom. In the interview, she said: *"We must achieve all the book objectives, because the student is eventually required to know everything in the book."* These objectives were as follows:

- Investigation of the relationship of the length of the triangle sides with each other.
- Investigation of the relationship of the length of the triangle sides with the measurements of its angles.
- Investigation of the properties of the isosceles triangle;
- Investigation of certain properties of the triangle.
- Knowledge of the outer angle of the triangle.
- Investigation of proving Pythagoras theorem on the right angle triangle.

The researcher noticed that she focused on these objectives through repetition and asking the students question in every lesson. This is emphasized by Hashweh (2005), who provided that the teacher should be aware of the general and special educational objectives, which he/she wants to achieve when teaching a certain subject. Also, the teacher should

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know why these objectives are important, in addition to his/her command of knowledge and skills the students will acquire eventually. This may explain the high results her students obtained in TIMSS (2015). This is also in line with Morris, Hiebert and Spitzer (2009) in that the teacher's awareness of the objectives and work toward achieving them positively reflects on the students' education and achievement.

2(B) Teacher (2): humble and contradictory knowledge of the general and special objectives.

Responses of Teacher (2) on the questionnaire items and the oral interview indicated that she has constructive objectives. She also believes in the role of mathematics in interpreting the natural phenomena, and indicated the need for linking math with the students' lives, since it is *"an inseparable science from our life"*. Nevertheless, these objectives were not reflected in her classroom practices.

As far the special objectives are concerned, the teacher was well familiar with them, which was clear in both the interview and classroom observations. She achieved all the prescribed objectives in the book, and said, when interviewed, *"Unfortunately, we are restricted to "give" the students the material that makes him/her get better grades."* The researcher found that the teacher was focusing on the objectives in her classes through repetition, and asking students question during the explanation.

3(A): Teacher (1): Wealthy and Broad Content Knowledge.

Hashweh (2005) and Shulman (1987), see that the teacher's possession of rich content knowledge is a basic requisite, so that his/her knowledge of how to teach the content will be good. In this sense, the researcher focused on detecting the content knowledge through different ways. These ways included: content-specific test, oral interview, classroom observations, certain documents, such as worksheets, examinations, to obtain an answer to the following question: "How far is the math teachers' knowledge of the pedagogical content concerning the "triangles" topic?"

This teacher was characterized by her broad knowledge as she obtained (47/50) degrees in the test. She is also quite aware of the vital significance of the content representations, which she employed to a wide extent in her teaching. And her representations were very meaningful and suitable for the content she teaches. The teacher feels the need to embody the intangible information to her students on factual grounds, to help them interconnect and remember the information. This finding is in line with Henze, van Driel and Verlhoop (2008) and Berry, Loughram and Mulhall (2008).

Teacher (1) adopted the perspective that science is integrative, inseparable, and she justifies linking "triangles" with other subjects, such as English language by mentioning the terms in English. She also sees that this aspect is very important, as she said, *"Math books are taught in English language in most of the universities; and in order not to create a knowledge gap with the students in future, we pave the way to them at present."* She also linked the unit material with physics, and underlined the significance of Pythagoras theorem in many of her lessons. She said:

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Without this theory, we were unable to record or download the digital music. This theory allows us to calculate the distance between two points. At school, we learn this thing on papers only, but in reality you can do this in the three dimensional (3D) models, such as calculating the diameter of the cube, for instance.

Although these facts are not included in the curriculum, she sees that this linkage is important to enrich the students' knowledge and change the lesson climate, which attracted their attention.

Teacher (1) feels the need for connecting the new information with the previous information. She paved the way for the unit by testing the students' previous knowledge, which she (herself) taught in the sixth and seventh grades, through asking the students oral questions, and organizing the explanation proportional to their responses. She said, *"Paving the way before them is very important, because if we start with the terms of the unit, the students will find the subject difficult."* For instance, she started the unit by testing the previous information with the learners by presenting the types of triangles, as shown in Fig. 2. Then, she asked her students, *"What are the types of these triangles?"*

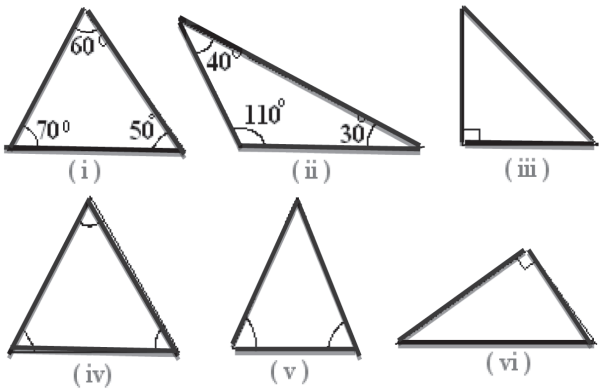

Fig. 2. Teaching Models the Teacher Utilized in her Teaching

On the other hand, Teacher (1) seemed not aware of science processes nor their importance for her, as this aspect was not apparent in the interview. Meanwhile certain science processes were embodied in her classroom practices. She asked her students to conclude the information and explain what happens in the educational videos presented to the students about Pythagoras theory, such as https://www.youtube.com/watch?v=G-w9iKERLonc&feature=youtu.be. Therefore, it seems that the teacher behaves this way building on her experience in teaching, not on her theoretical background.

When the researcher asked her about the important ideas in the unit, she answered, *"All the unit parts are important".* She stresses on everything in the unit, and explains it in details. She is careful to make her students get good results, in addition to enriching

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their minds with new knowledge through navigation in the unit contents and linking the material to their lives.

We can conclude from the above statements that the teacher has good knowledge about the content, particularly in the aspect concerning its representations, and linking the math with other subjects, which will be reflected on the students' exam in TIMSS (2015). This is in line with the study of Ng (2012); Beswick (2007) which showed that the use of representations to a wide extent in mathematics, and ability to alternate between them are strongly connected with the success in math teaching and learning. In addition, the ability to interlink between the different mathematical ideas and concepts is important to understand these concepts. It is important to encourage the student to see these interrelations among the concepts through discovering different representations of the concept. Study of Hwang, Chen, Dung and Yang (2007) showed that the students' skills in using the multiple math representations are their keys in math problem-solving.

3(B) Teacher (2): Content Knowledge is less than the desired level.

Content knowledge of Teacher (2) was low, as she obtained (40/50) degrees. When asked through the interview about the reason for her mistakes, she ascribed it to her inability to look into the test items accurately. On the other hand, there was a contradiction between the oral interview and the classroom observations about content representations that fit the unit. She considers content representations an enrichment material added to the course material, not a basic material, and should not be planned or prepared. She also indicated through the interview that she did not plan or prepare the lesson all the times, and said: *"Sometimes, one sees herself an experienced teacher to the extent that she overlooks planning, preparing, or the like."* Even though, she employed some representations.

The interview also showed that her ability to interrelate among the materials is limited. The researcher sees that this is natural as the teacher focuses on high degrees that her students will get in the exam, which her performance evaluation (by the school principal) will be built upon. This encourages her to limit herself to explain the book information clearly and in details. In this concern, she said, *"I am concerned with the quantity rather than the quality in the material, my goal in this unit is to explain the information contained in it completely (100%) to enable the students obtain highest degrees."*

In the same vein, she indicated in the interview that she could not interconnect between the concepts and information stated in the unit, on one hand, and the previous concepts and information with the students, on the other. She sees that the 7th grade material is 1% related to the unit material, and that all the unit contents are new for the students. Nonetheless, this was not reflected in her classroom practices, as she prepared for the unit through testing the previous information with the students, which she taught them in the 7th grade, by asking them oral questions and building explanation on their responses. The teacher drew the triangles, and then she asked her students to tell their types. When she was asked about the important ideas in the unit, she answered: *"All the unit parts are important."* She accurately explains what is stated in the unit.

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4(A): Teacher (1): Distinguished knowledge of the vertical and horizontal integration of the curriculum.

Teacher (1) possesses rich knowledge in the curriculum and integrated it vertically, through linking the unit information with those the students learned in the 6th and 7th grades. This was clear in asking oral questions to the learners, listening to their responses and then commenting on them. It was important for this teacher to ensure that the students' knowledge include the triangles concept. She discovered that some students did not have sufficient knowledge that enabled them provide a correct definition of the triangle. Leena (female student), for instance, defined it as a three-side model, and the student attempted to improve her knowledge by drawing a triangle on an idea-presentation paper. Then, she asked a number of questions which led Leena to differentiate between the model and flat surface that the paper represents. The following box is an example of the dialogue between both:

Teacher	Who can tell the triangle definition?
Leena	It is a three-side closed model.
Teacher	When we drew the triangle, where did we draw it? Is the paper a model or a flat surface?
Leena	Flat surface.
Teacher	Okay, what's the meaning of the flat surface, Leena?
Leena	She made a facial sign that she didn't know.
Teacher	The model is the figure that has three dimensions: length, width and height, such as the pyramid and cube.
Leena	Ahhhh! I understood.

As for the horizontal integration, which is concerned in finding the horizontal relationship among the different areas of the curriculum, the teacher linked the unit material with other materials. It made this linkage by explaining the math relation upon which the scientists depended in building the pyramids and temples, road planning, constructing the buildings, engine manufacture, designing the display systems such as TV, furniture manufacture, designing and planning different playgrounds according the rules of the different games, and calculation of geographical distances.

The teacher added that the curriculum is "intensive" and contains "heavy" information, and the lessons quota is insufficient to cover them all, which necessitates us give additional lessons. This is an indication that the teacher has wealthy information about the curriculum, which was positively reflected on her students' performance in TIMSS (2015). In this regard, Berry, Louphran and Mulhall (2012) see that the good teacher is the one able to enrich the curriculum, both vertically and horizontally, and tackle it to match his/her students' abilities and characteristics.

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4(B) Teacher (2): Limited knowledge of the curriculum vertical and horizontal integration:

The response of Teacher (2) in the oral interview contradicted her classroom practices. She said that there is only 1% connection between the unit material and the materials already studied in previous grades. Despite this, her classroom practices showed that she enriched the curriculum vertically, by asking her students a series of oral questions, listening to them and providing comments. These questions revealed their previous knowledge. As for the horizontal enrichment, she simply connected the unit content with other subjects, such as English language, which appeared through pronouncing certain terms of the unit in English.

The teacher said that the curriculum is demanding and has great amount of information. She added, *"The unit is actually intensive, every line has an idea, and our lessons quota is insufficient to cover them, which mandates us give additional lessons."* She added that many important information were deleted from the new curriculum (2015 edition), and were replaced by main headings without details, which required her explain the deleted information by clarifying to the students in details, as were in the old curriculum. The above paragraphs ensure that she has limited and contradictory information about the curriculum, which is in disagreement with the findings of Hashweh (2005) that the good teacher is the teacher who is able to enrich the curriculum, both vertically and horizontally, and process it to fit his/her students' abilities and characteristics. Perhaps, such contradiction negatively reflected on her students' performance in TIMSS (2015).

5(A) Teacher (1): Wealthy knowledge of the learning resources:

This teacher believes in enriching her knowledge through multiple resources. She said, *"I attempt doing so through the internet, in spite of the fact that our time is very compacted. I have to prepare video films, and if I find something interesting, I have to bring to my students, because the subject sometimes looks very rigid. I feel they need to see video that may better explain the idea. Sometimes, I watch good things that are pertinent to my curriculum on the TV, and tell my students follow these TV shows. Moreover, sometimes I look up in books from a library."* This view of the teacher is in agreement with the results of Wake and Pampaka (2008), which stress the importance of the external and internal resources to support the teachers' practices. This was clear in her classroom practices, as she employed video clips, illustrative models and PowerPoint. She also used GeoGebra program. Figure (3) shows some of the students' drawing to categorize the triangles as per the length of their sides.

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Equilateral triangle	Isosceles triangle	Scalene triangle

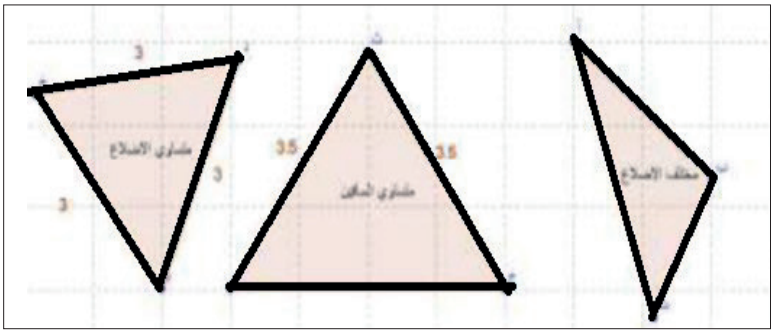

Fig. 3. Shows some of the students' drawing to categorize the triangles as per the length of their sides

The teacher further sees that the use of such means widely affects the students' learning. They help in communicating the information to her students, "change" the lesson atmosphere, break the routine, and assist in attracting their attention. Furthermore, she believes that such means help her students remember the educational material to a wide extent, which positively reflected on the performance of her students in TIMSS (2015). These findings are in line with the studies of Baumert, Kunter, Blum Brunner, Voss, Jordan and Tsai (2010).

5(B) Teacher (2): Low knowledge enrichment from different resources:

The researcher found that Teacher (2) was unaware of the importance of enriching her knowledge from different resources. On the other hand, she had a negative point of view toward these resources. She said that: *"The use of educational programs from the internet in the classroom may hinder teaching."* Although she believes in their importance, she contradicted herself in another point in the interview. She said, *"You cannot rely on the book only; the teacher who depends on the book only is equal to the student; any student who brings "external" information shall outperform his/her teacher."* This view made her in disagreement with the results of Hashweh's Model (2005). Her negative view toward the learning resources was clearly reflected in her classroom practices. She did not employ any external teaching resource, and confined herself to the blackboard, color pencils, and the schoolbook when she taught her lesson, which also negatively affected the students' attention. In this concern, the researcher noticed, while in her classroom, that the learners showed boredom and attention dispersion,

This had impacted the results of her students in TIMSS (2015), as their results were better in her first year in service than the years after. She added that she enriched her knowledge through many resources, such as use of video and educational flashes from the internet, and consulting the experienced teachers in case she faced any problem.This is quite in line with Baumert et al. (2010).

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6(A) Teacher (1): Modest knowledge of the context:

Pursuant to the analysis of the oral interview and classroom, it was clear that the teacher's knowledge of the context was somehow limited. She was interested in building good relations with her students, which go beyond the school borders. She said, *"My relations with my students are excellent and remain so even after the school time; they inquire about me at the social level, because this is helpful in teaching. If the teacher is "rigid" and authoritarian, the student will fear dealing with him/her; always attract the students rather than make them feel afraid of you and the subject to be taught. How would the student understand the material "if afraid"?"* She further added that it is quite important to understand the students' social conditions, and attempt to encourage them overcome the problems they face in their practical and educational lives. The researcher noticed that this aspect was not apparently clear in the classroom observations.

On the other hand, the teacher's knowledge of the general educational system was not sufficiently clear, such as number and duration of the lessons allocated for each unit, nor was her knowledge of the environment that surrounds her students. She also did not talk about her relationship with the teaching staff during the interview, which could be attributed (in the researcher's point of view) to her unawareness of the educational context concept. However, the good relations with the teaching staff appeared at a modest level during our classroom observations, when the computer teacher assisted the interviewed teacher in preparing the computer lab to enable her give the lesson; the secretary also assisted the teacher in changing the color pencils for the students in the groups; and, the teacher cooperated with the principal and other teachers to coordinate the lessons and alternate them to fit the researcher's time.

6(B) Teacher (2): Modest knowledge of the context.

This teacher showed only a limited knowledge of context, as she cares for building good relations with her students for their positive effect on the teaching process. She said, *"My relations with my students are excellent, we tell each other jokes, but within certain constraints. They serve me in teaching when I ask them to attend an additional lesson; they all come."* She also places much attention to the achievement level of her students, and justified this attention by that "The low achievement level requires her to increase the lessons allocated for the unit." This also appeared through the classroom observations. She was very interested in building good relations with her students, but the researcher was unsure of her interest in the social aspects, and faced difficulty to detect it. Probably, this may be attributed to that the teacher did not perceive the educational context meaning, which made him explain it by offering some examples.

On the other hand, the teacher was aware, to a wide extent, of the general educational system, such as number of the lessons allocated for the unit. The researcher could not perceive her interest in the environment that surrounds the learners, and her relations with the teaching staff. But, he could judge the relation between her and the teaching staff while he was inside the school, where there was an atmosphere of love and familiarity

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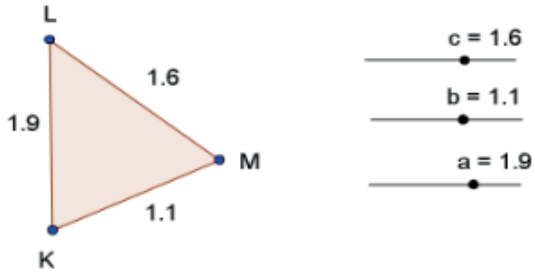
among them, which was clear in their discussions and organizing the lessons the way that fits the times of the researcher.

7(A) Teacher (1): Wealthy knowledge of the teaching strategies:

This teacher was characterized by using a variety of meaningful educational strategies which fit her students. She believes in the important role the educational strategies play in teaching, and their wide effect on the students' learning. This is in line with Voss, Kunter and Baumert (2011) who see that simulation helps in detecting the misconceptions of the students, and plays a vital role in rebuilding the conceptual framework with them. In addition, these strategies help the students in understanding the mathematical concepts. Figure 4 shows the shape of the dialogue between the teacher and one of the students.

Teacher: we can group the triangles by the lengths of the sides. Let us see the Geo-Gebra.
Teacher: look at this triangle, how do you see its sides?



Sara: the sides are unequal (of different lengths)
Teacher: then, what can we call this triangle?
Amal: scalene triangle.

Fig. 4. A dialogue between the teacher and one of the students

The teacher asked her student's oral questions, sequentially, to build her explanation on their responses. She also focused on raising dialogue and discussion in the lesson between her and her students, as well as among the students themselves. She made comments on their answers through the feedback, or linking the information in hand to previous information. Furthermore, she offered video clips that explicate the triangles properties such as: https://youtu.be/mLeNaZcy-hE, which attracted the attention of her students. Sometimes, her students asked her to replay the video, and she accepted their request with welcome. In addition to employing drawing in her teaching, she asked her students to find out the number and types of triangles that can be drawn within Figure 5.

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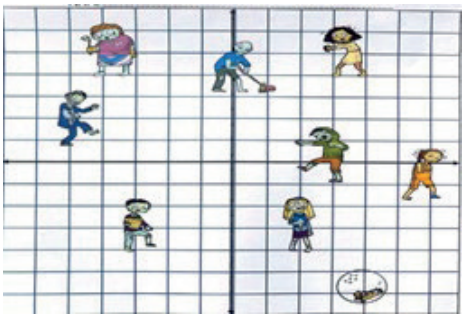


Fig. 5. The model the teacher employed in her teaching

She applied the group strategy in her teaching to assess her students' understanding of the material she explained to them. She distributed her students into five heterogeneous groups, and distributed the worksheets to them. The questions of every group were different from the others, as shown in Fig. (6); she discussed them with the group members. She added that she would ask questions to any of the students in the same group, to ensure participation of all them in the cooperative work. Meanwhile, she was moving around among the students explaining the questions to them, if requested by any of them. She asked every group to display the work on the blackboard with explanation of the method of answers. She also warned them not to fall in mistakes during solution; her students did not show any misconception.

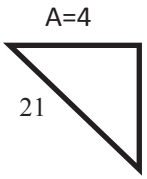
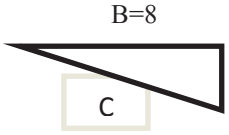
(C) Find out whether (or not) the following lengths form a right angle triangle: (1) $16^2 = ? 12^2 + 8^2$ $256 = ? 144 + 64$ 256 (does not equal) 208. Then, it is not a right angle triangle. (2) : 7, 24, 25 $7^2 + 24^2 = ?25^2$ i.e. 49 + 576 = 625 The above lengths form a right angle triangle.	Find out the length of the unknown side, round the solution to the nearest part of 100, if any.  $C^2 = A^2 + B^2$ $21^2 = 4^2 + B^2$ $441 = 16 + B^2$ $Ö425 = Ö B^2$ $B^2 = 20.62$	Find out the length of the unknown side, round the solution to the nearest part of 100, if any.  **$C^2=A^2 + B^2$** **$C^2= 6^2 + 8^2$** **$C^2 = 36 + 64$** **$Ö C^2 = Ö100$** **C = 10 Length of the side is 10**

Fig. 6. Examples of the students' works in the groups

This teacher sees that diversifying the teaching methods and strategies "breaks down" the routine and increases the students' motivation to remain "in touch" with

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the lesson. She said that she *"Prefers the representation method, because it embodies the information; and that applying the group method strengthens the love spirit and develops the leadership spirit among the students."* In addition, she takes into consideration the individual differences among her students, and removes the barrier of shame and fear with some of them. She usually discusses the disputes and quarrels that occur between her students by putting them in the same group, to develop the spirit of cooperation. The teacher attempted her best to develop the skills of her students by raising discussion and dialogue throughout the lesson time; thereby revealing their previous knowledge and testing their understanding of the explained material. By distributing them into groups, she pushed them to think in the questions, approach solutions and display them before their colleagues in the classroom, to assess the extent of realizing the desired objectives. She offers her students opportunity to think in the questions, discuss them, and conclude the information by their own. She said, *"I like to hear from the students while thinking in the question, or in groups, then I give them the typical answer, because they are required to obtain the typical answer, which the Ministry of Education provides, and which their answers will be corrected according to it, but I do not give it to them directly."*

As for the evaluation, the teacher assesses her students continuously in the class. She applied several methods, such as: oral questions, worksheets, projects, daily and final examinations, taking into account that they should be varied to match thinking levels of the students, because she believes in their individual differences. During the interview, she showed interest in the misconceptions and learning disabilities they may face. Once assuring their presence, she intentionally questions the students about them in the exams, to ensure that they were treated. Figure 7 shows examples of the students' works.

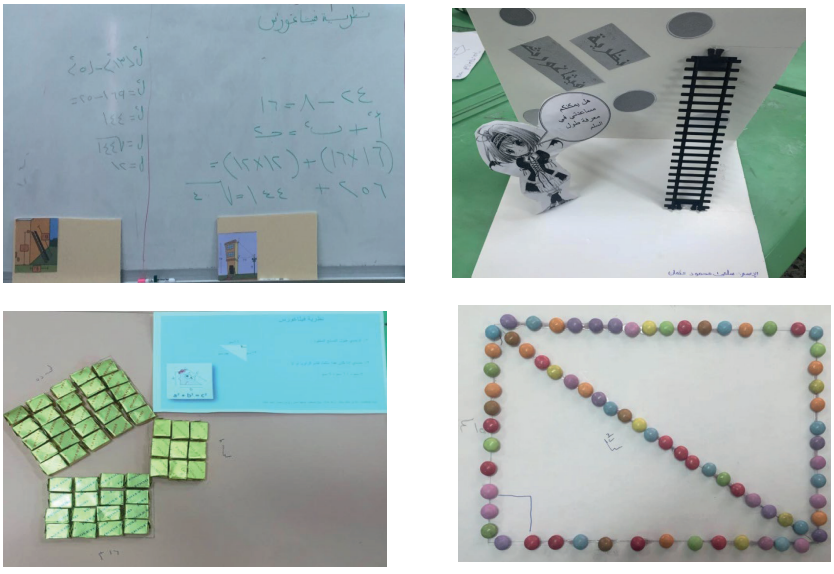

Fig. 7. Examples of the Students' Works

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The teacher did not focus on the external activities and research about certain subjects, such as homework given to the students, due to the time tightness. When she was asked about the nature of the duties she gave to her students, she answered, *"I may give them extra questions as homework, such as questions stated in TIMSS, questions of the book, or else if a student asks about something, I may ask them to look for it, and we discuss it in the next lesson"*, which reflected positively on her students' performance in TIMSS (2015). She also checks the written answer literally, as she advised her students in one occasion, saying, *"You are going to choose the scientific stream, you must write everything in details. You must express your thoughts correctly"*. She saw that the degrees of the question should be distributed over them fairly, to be corrected fairly. These results are in agreement with those of Fauskanger and Mosvold (2010), which showed that the constructive learning strategies include teaching activities oriented to math teaching, where the exploratory learning, problem-solving and group learning occur.

7(B) Teacher (2): Medium knowledge of the educational strategies:

The researcher noticed that this teacher could not employ the constructive scientific methods, and believed that the group method is a time waste without achieving the desired objective. As for the worksheets, she saw that they did not have a positive effect on the students' learning, because they depend on each other in solving the questions. This is quite in contradiction with her constructive beliefs that she stated in the questionnaire, in which she indicated the effect of the social interaction and dialogue among the peers on the learning.

On the other hand, the teacher said that it was necessary to avoid the lecture method during teaching the unit. Even though, her teaching method was limited to the lecture style, and lecture with discussion to explain the unit. Contradiction was clear between her beliefs and what takes place on the real ground. She considers the illustrative methods very important part in teaching, which considerably affects the students' learning, and moves the student from one learning style to another. She gave examples such as, flashes, video, colors. However, unfortunately, this view was not clear in her classroom practices when she was limited to the color pencils and blackboard.

Another contradiction appeared in her attempt to develop her students' skills, which was clear in the classroom practices, but the interview did not include them. She sees that the group teaching is a time waste, and the lecture is the best method to teach the unit. Therefore, the teacher (in her view) is the axel of the educational process and the student's role is eliminated. Subsequently, she contradicted her constructive view on the role of the learner-teacher, which she referred to in the questionnaire and the interview too. Quite the opposition, she attempted to develop the skills of her students in her lessons through raising dialogue and discussion between her and them throughout the whole lesson. She aimed at revealing their previous knowledge and testing their understanding of the material; or by offering them certain questions that motivate them to think and conclude.

She had good knowledge in evaluating her students. She employed instruments to identify her students' understanding, such as: asking oral questions, external questions, and daily and final exams, which were based on her belief in the individual differences among the students.

In the same context, the researcher saw that the teacher didn't focus on the external activities or research works about certain subjects, such as homework given to the learner, which she justified by the *"tightness of time, because the student is required to know the information in the book only." She added, "Therefore, the exam reduced the educational process efficiency, which made the student a machine to store information, and the teacher intensively focuses on ways that increase the success rate of the students, regardless of the way and method applied."* When she was asked about the nature of the homework she gave to her students, she answered, *"Most often worksheet, thinking in the book solutions, one external question to think in; and, the students are not required to solve all the questions of the book"*, which negatively reflected on her students' performance in TIMSS (2015). This is in line with the studies of Hashweh (2005) and Kroll (2004), which showed that teaching for the constructive teachers is not only in solving algorithms, but training is also equally important, although not sufficient to understand the basics. The study results showed that learning for the experimental students is solving algorithms, limited solution styles, and training on problem solving.

8(A) Teacher (1): Good knowledge of the students' characteristics

The results showed that this teacher holds a belief that the learner has the ability to develop his/her thoughts and use them to assimilate the new ideas; and that the best math teaching methods require facing the students' misconceptions. This was clear in her classroom practices as she was frequently asking the students oral questions to build explaining the lesson upon their responses, through linking the previous knowledge with the new. Thereby, she was able to unveil the misconceptions with the student at the same time. Figure 8 shows the dialogue between them.

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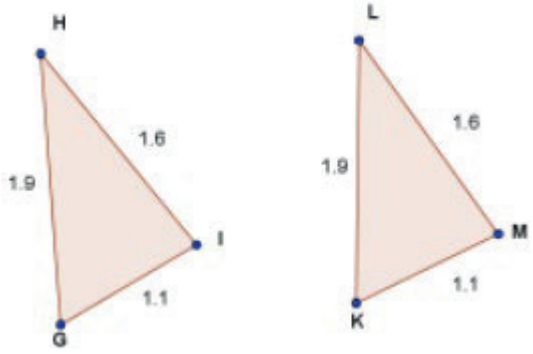
Teacher: pay attention to Geo-Gebra and look at these two triangles… what about them?



Salma: the triangles are of the "same size"
Teacher: what does it mean they are of the same size?
Leen: it means that the lengths of the sides in the first triangle equal the lengths of the sides of the second triangle, the sides are equal.
Teacher, well, students, since the sides of the first triangle are equal to those in the second, are the triangles congruent?
Huda: yes, they must be congruent.
Teacher: we want to make sure, how?
Leena: we move the first triangle and place it over the second. (Salma moves the triangle). Here are they, they are congruent.
Teacher: okay, what is the condition for congruence here?
Aya: they are congruent based on "equality of the three sides" rule.

Fig. 8. The dialogue betweenteacher and students'

The teacher focuses on knowing the students' social conditions, or anything that may affect them, and has no concern about their previous academic achievement. When the researcher asked her about this, she said, *"I don't have to build on any previous thing, because this may leave undesirable impression with the student. I do not care for whose marks are within 90s or who ranked first in the class. I deal with all of them equally, and later I discover everything."* She further added that the class includes the three academic levels (excellent, very good, and good), with variation in the percentages of each level; and that the number of the students within the good level is low as compared with the other two levels.

Many of the previous studies indicate the importance of the teacher's knowledge of the characteristics of his/her students, so that he will be aware of the environment that emerges during the learning process, whether classroom or local environment; and that the teacher should have ability to utilize the resources of the environment and employ them to serve the teaching process (Jutner & Neuhaus, 2011).

This result is in line with the constructivism theory idea in that the learners build their knowledge on the previous learning (Langrall, Alagic, & Rayl, 2004). To sum the knowledge of Teacher (1) of how to teach the eighth graders the triangle unit content, her strengths were apparent in; curriculum, learning resources, teaching strategies, and educational content. Yet, her knowledge varied in the beliefs, educational objectives, educational context and students' characteristics. Her strengths might have the positive effect on her students' performance in TIMSS (2015).

8(B) Teacher (2): Limited knowledge of the students' characteristics:

Through the responses on the questionnaire items, the teacher showed her belief in that the learner has the ability to develop his/her ideas and use them in assimilating the new ideas; that the best math teaching methods require facing the students' misconceptions. The teacher also is interested in the learners' academic level, but she does not attach any importance to their social conditions. She indicated that the class includes the three academic levels (excellent, very good and good), and their rates vary, but majority are grouped within the good and very good levels.

Through the classroom observations, the researcher found that she does not care for selecting her teaching method to match the levels of her students. She limited herself to the lecture method and discussion, without employing any other educational methods or illustrative aids. She also indicated that she diversifies her oral or written questions to allow all the students chance to participate in the class, given that there are individual differences among them. These differences appeared when she taught them in previous years, or through their interaction in the class, the oral questions asked during the teaching process, the distinguished questions, or even through the features of their faces. In the same context, the teacher, during the interview, said that the triangles unit is suitable for the students, but it needs more preparation and clarification.

Based on the above, it seems that the teacher's knowledge of how to teach the content varies, but it was close to being poor knowledge. Her knowledge was strong on the objectives and teaching strategies, but modest in the educational context, beliefs, pedagogical content, curriculum and learning resources. Most probably, this made the negative effect on her students' performance in TIMSS (2015).

Conclusions

In the light of the abovementioned results, perhaps we can explain the reason of the outperformance of Teacher (1) students over those of Teacher (2) in TIMSS test, by that the way the content is taught affects the students' performance. Therefore, the researcher recommends carrying out a study to explore the relation between the teachers' practices and classroom practices in mathematics, compare them with the results of this study, and utilize the resources where the teachers acquire their beliefs, in an attempt to change

their beliefs toward constructivism. This is vitally important because all the beliefs have roots and sources related to them, such as training courses held by the Ministry of Education, educational supervisors. We should consider the self-reflection in the experiences and the need for highlighting the significance of the beliefs the teacher holds about the learning process nature. Particularly, the previous knowledge with the learner, enhancing brainstorming, challenging the students' thoughts and underlining their limitation in terms of the scientific thoughts. This could be embodied by the Ministry of Education. The educational supervisors are also required to focus on the teachers' beliefs about learning, when they assess the teachers, and look for the sources of these beliefs to attempt providing the delicate, remedial and developmental methods. The supervisors should work toward the constructive thought based on the meaningful dialogue and collective work, to upgrade the teachers' community to be of higher efficiency; introduce the element of mediation and self-critic in the daily planning book. All these will assist the teachers develop themselves through knowing what they had done in the academic year; and will avoid enhancing routine in their work, which could be reflected in their beliefs about learning.

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Matematikos mokytojų žinios, kaip mokyti aštuntos klasės mokinius trikampių temos

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Santrauka

Straipsnio tikslas – išnagrinėti 8 klasės matematikos mokytojų žinias apie tai, kaip mokyti trikampių temos. Šiuo tikslu buvo taikomas Hashweho modelis (2005). Modelis apima šiuos komponentus: mokytojų pedagoginio turinio žinias, įsitikinimus apie mokymąsi ir mokymą, besimokančiųjų savybių žinojimą, edukacines strategijas, švietimo aplinką, mokymosi šaltinius, ugdymo programą ir turinio bei filosofijos aiškumą. Tyrime dalyvavo mokytoja, kurios mokiniai gavo aukščiausius įvertinimus Tarptautiniame matematikos ir gamtos mokslų gebėjimų tyrime (angl. *TIMSS*, 2015), ir mokytoja, kurios mokiniai gavo žemiausius šio testo įvertinimus. Tyrėjas taikė kokybinį metodą, reikalingą duomenims surinkti, naudodamasis įvairiomis tyrimo priemonėmis: klausimynu, interviu, pamokų stebėjimas, turinio testas ir dokumentų analizė. Rezultatai parodė, kad mokytojų žinios apie tai, kaip mokyti turinio, skiriasi. Mokytoja, kurios mokiniai gavo aukščiausius TIMSS (2015) įvertinimus, geriau išmanė šiuos aspektus: pedagoginį turinį, turėjo savo įsitikinimus apie mokymąsi ir mokymą, besimokančiųjų savybes, edukacines strategijas, švietimo aplinką, mokymosi išteklius ir ugdymo programas. Kita vertus, abiejų mokytojų žinios apie ugdymo tikslus buvo vienodos.

Esminiai žodžiai: *pedagoginės žinios, konstruktyvizmas, matematiniai pasiekimai, mokytojo praktinė veikla pamokoje, Tarptautinis matematikos ir gamtos mokslų gebėjimų tyrimas (angl. TIMSS).*

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