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Collaborative Problem Solving in Mathematics: A Systematic Literature Review

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Annotation. This study aims to explore the definition, task characteristics, and themes in studies of CPS in mathematics. An SLR was conducted on articles from 2002-2022 using Scopus database. The thematic analysis identified five themes of studies: Technology, Discourse analysis, Social Skills, Mathematical thinking, and Students' achievement. Research gaps and potential future studies within each theme were explored.

Keywords: *collaborative problem solving, mathematics education, systematic literature review, trends.*

Introduction

Collaborative problem solving (CPS) has become a focus following the need to improve students' skills in the 21st century. CPS, which contains two essential 21st-century skills, i.e., problem-solving and collaboration, has been discussed and defined by various entities. PISA defined CPS as "an individual's capacity to effectively engage in a process in which two or more agents seek to solve a problem by sharing the understanding and effort required to reach a solution and pooling their knowledge, skills, and efforts to reach that solution" (OECD, 2017). The need to investigate students' CPS had been seen from the 2012 PISA focus. The focus was on interactive problem solving, where students faced problems and required them to interact with tools or media to obtain adequate information (OECD, 2013). It was then changed into CPS in PISA 2015 (OECD, 2017).

Another project focusing on CPS is the Assessment and Teaching of 21st Century Skills (ATC21S) project. The project defined CPS as "approaching a problem responsively by working together and exchanging ideas" (Griffin & Care, 2015). Further, it stated that CPS is a joint activity in which a group carries out several steps to turn a problem condition into a desired goal. While PISA gave a content-dependent explanation of CPS, ATC21S divided CPS into content-free and content-dependent categories. The content-dependent CPS involved skills and knowledge of CPS in particular content such as mathematics and science.

In the context of mathematics teaching and learning, there has been a massive growth in studies investigating CPS. The studies had various focuses, and each focus was explored to some extent. For example, a study by Stacey (1992) analyzed whether group problem solving was better than individual problem-solving. Interestingly, the individual performance was better than group because students tended to accept peers' ideas without asking for justification. Similar studies were conducted comparing individual and group problem solving (Barron, 2000; Kapur & Bielaczyc, 2012; Schmitz & Winskel, 2008). Other studies investigated an effective way to assess CPS in mathematics (Chan & Clarke, 2017; Harding et al., 2017) or analyzed teaching-learning practices to improve students' CPS (Chiu, 2008a; Hähkiöniemi et al., 2016). Besides, it was shown that studies on CPS characterized CPS tasks differently.

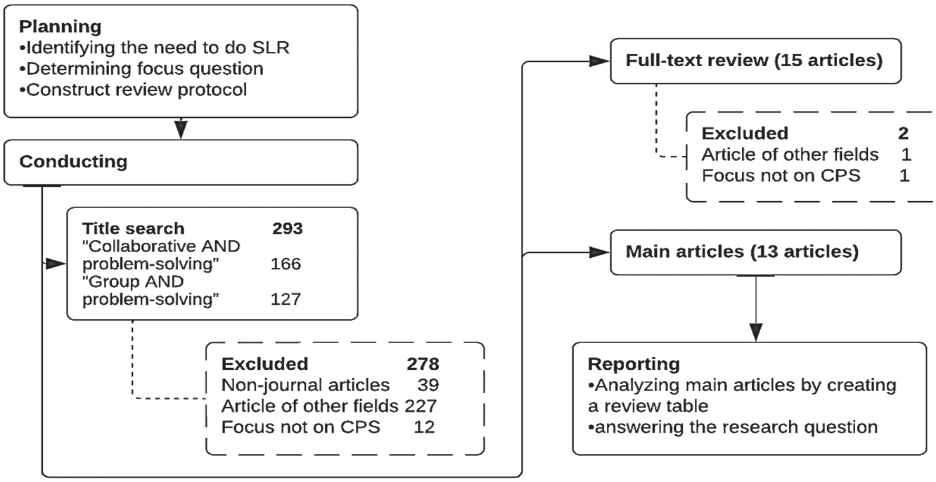
Considering numerous studies about CPS had various focuses, it is necessary to collect and better understand the study findings of CPS in mathematics. Thus, this paper aims to do a systematic literature review on studies about collaborative problem-solving in mathematics for the past 20 years. More specifically, this paper will answer two research questions: (1) What is collaborative problem-solving in mathematics, and how are the characteristics of tasks used for CPS? and (2) In what themes are studies on CPS in mathematics being conducted? The result of the SLR will benefit the current understanding of CPS in mathematics and how far each research focus has been investigated.

Method

This study used a systematic literature review as its method. A systematic literature review is a review of the available studies or literature using explicit, accountable, rigorous research methods (Oakley, 2012). Specifically, this is an adaptation of a guideline by Dodd et al. (2017). The guideline contains three phases: planning, conducting, and reporting, as seen in Figure 1. The focus of this SLR is to review studies of collaborative problem-solving in the context of mathematics education.

Figure 1

Phases of SLR Adapted from Dodd et al. (2017)



After establishing the need and the focus question, a review protocol was constructed (see Table 1). For the title search, the Scopus database and articles published during 2002-2022 were extracted in May 2022. The search keywords used were "collaborative AND problem-solving" (166 titles retrieved) and "group AND problem-solving" (127 titles retrieved). The latter keyword included studies that utilized "group problem solving" as their terms (e.g., Smith & Mancy, 2018). The 293 titles were input into a software, namely *VosViewer*, to visualize the clusters of the studies and how they were related. An overlay visualization of those studies was also created to show the trends from 2002 to 2022.

Two hundred ninety-three titles underwent an initial screening by reading their title and abstract and identifying the publication type. All titles of books, thesis, proceedings, or non-journal articles were excluded. Additionally, studies that did not represent research in mathematics education or mathematics were excluded (for example, a title of study on CPS of employees in a particular company). During the abstract reading, the focus was to include articles that specifically investigate CPS in a mathematics context and exclude articles that were outside of this focus. Articles investigating CPS in other subjects (e.g., science) or investigating individual problem solving were excluded. Fifteen articles were obtained from this stage. After that, the fifteen articles' full text was carefully read to determine the main articles. Two articles were excluded, resulting in thirteen articles as the main articles.

Table 1
Inclusion and Exclusion Criteria

Inclusion criteria	Exclusion criteria
Articles published on 2002–2022	Articles published before 2002
Written in English	Written in other languages
Peer-reviewed journal articles	Proceedings, books, proposals, thesis papers, or other non-journal articles
Empirical studies in mathematics learning or mathematics education	Studies on other fields or literature reviews
Focus on CPS or in the CPS context	Others (e.g., individual problem solving or collaborative activities without problem-solving)

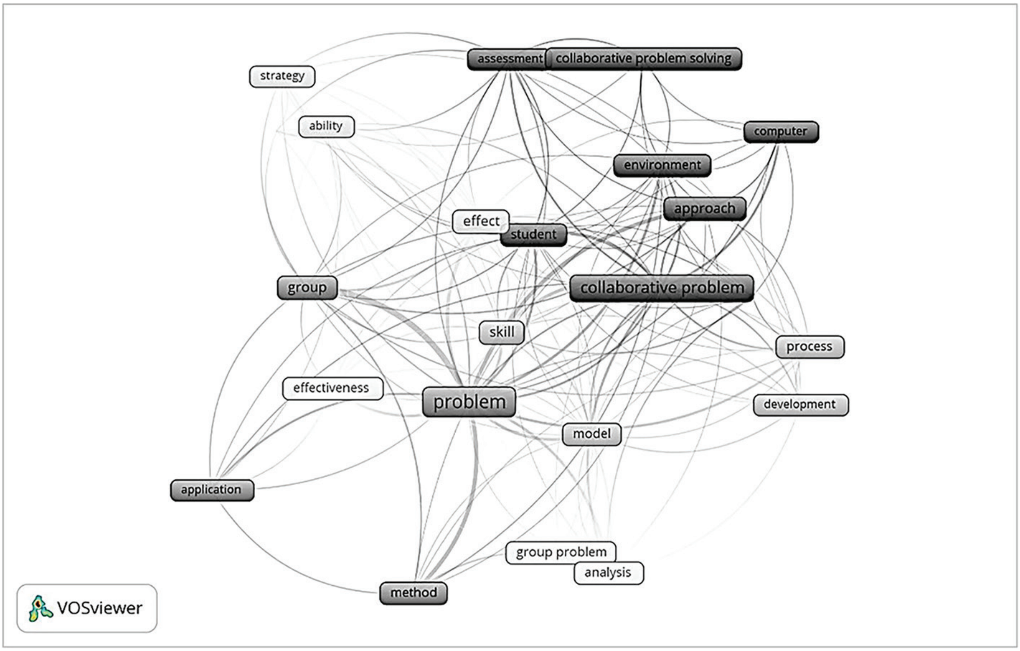
The main articles were analyzed in the reporting phase by creating a review table after identifying the main themes, methods, results, and recommendations. One article might belong to more than one theme. The theme informed the variety of focus those studies of CPS had. The results of the main articles could inform current knowledge on CPS, while the recommendation might suggest future studies on CPS. Specifically, to answer the first research question, besides extracting the theories incorporated from the main articles, theories from two books (Griffin & Care, 2015; OECD, 2017) were added to support the theoretical foundation.

Results and Discussion

This section elaborated on the overall trends of studies of CPS, the characteristics of studies in the main articles, and the discussion to answer the research questions. The overall trends of studies in collaborative problem solving from the 293 titles obtained from the title search were displayed utilizing a software named *VosViewer*. The map of clusters of studies obtained from 293 articles can be seen in Figure 2. There are five clusters, each was shaded in a different grey gradient, displayed on the map: cluster 1 (collaborative problem solving, collaborative problem, computer, environment, role, student, approach assessment), cluster 2 (problem, group, method, application), cluster 3 (development, model, process, skill), cluster 4 (ability, effect, strategy, students' problem), cluster 5 (analysis, effectiveness, group problem).

A frame represented each word in the cluster, and frames were connected by an edge/link. The bigger the word frames, the more often they appeared within the articles, and the closer the frame to another frame, the more related they were (van Eck & Waltman, 2022). It showed that "collaborative problem solving" was closely related to words in cluster 1 and the word "ability" in cluster 4. By observing the frame size, the term "collaborative problem solving" did not appear quite often compared to other more general terms such as "problem" and "collaborative problem". It was understandable as some studies used several other terms in place of CPS, e.g., group problem-solving.

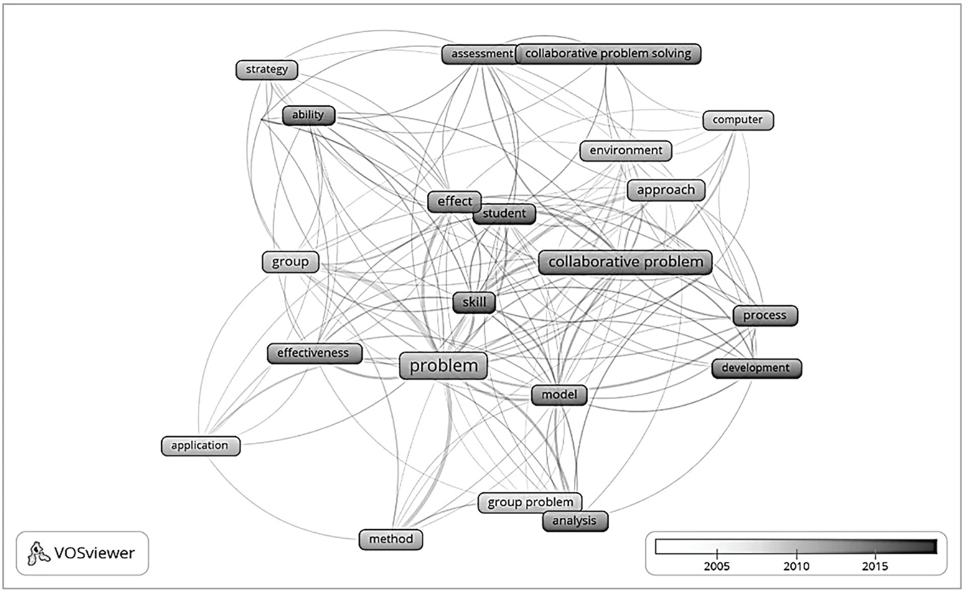
Figure 2
Clusters of Studies Investigating CPS



The overlay visualization of studies on CPS is shown in Figure 3. The map could inform the trends of studies over time. The studies related to group problem solving, identified by the term "group", "problem", and "group problem", were conducted before 2015 (shown by a lighter grey colour). The term "collaborative problem solving" was shown in grey, informing the use of the term in the studies starting around 2015. The dark grey colour for the terms "student" and "skill" informed the likely current trends in studies of skills related to CPS. The coloured versions of Figure 2 and 3 are available upon request to the corresponding author.

Figure 3

Overlay Visualization of Studies on CPS



Studies characteristics of the main articles

Study characteristics are shown in Table 2. Most of the articles selected were published on 2007–2011. Most studies investigated secondary students, and the most frequent study design was qualitative. Some studies utilized tasks in a single mathematics topic; some used various tasks with more than one mathematics topic, while other did not specify their topic.

Table 2

Study Characteristics of the Main Articles

Studycharacteristics	**n**	**%**	**Studycharacteristics**	**n**	**%**
Period			Studydesign		
2002–2006	2	15.4	Mixedmethod	4	30.8
2007–2011	5	38.5	Qualitative	6	46.2
2012–2016	2	15.4	Quantitative	3	23.1
2017–2021	4	30.8	Mathematics topic*		
Participantslevel			Algebra	3	23.1
Elementary	3	23.1	Geometry	4	30.8
Secondary	5	38.5	Function	2	15.4
HigherEducation	3	23.1	Probability	1	7.7
Adultsorteacher	2	15.4	Arithmetic	4	30.8
			Notspecified	2	15.4

*) The sum of the count was not equal to 13 as the categories were not mutually exclusive

Of the thirteen articles selected, two articles were published in the Journal of Mathematical Behavior, while the others were each published in the following: Journal of Educational Psychology, British Journal of Educational Psychology, Journal of the Learning Sciences, European Journal of Teacher Education, Journal of Computers in Education, International Journal of Computers for Mathematical Learning, British Journal of Educational Technology, Journal of Experimental Psychology: Applied International Journal of Computer-Supported Collaborative Learning, International Journal of Innovation, Creativity, and Change, and International Journal of Mathematical Education in Science and Technology. The main articles' findings were summarized in Appendix.

Definition and task characteristics of CPS in Mathematics

Two main topics were discussed in understanding CPS in Mathematics: definition and task characteristics of CPS in Mathematics. We believe it is important for each topic to discuss CPS in a general context and then narrow it down to CPS in mathematics. For CPS definition, we found two perspectives emerged from the two books and thirteen main articles: when CPS was considered a skill and when CPS was seen as an activity. For the task characteristics, three main characteristics of the task were identified.

Definition of CPS in Mathematics

Several works of literature have defined CPS as either competency or the activity of solving a problem together. PISA 2015 defined CPS as an individual's capacity where two or more individuals engage in a process to solve a problem. PISA defined CPS by stating that CPS competency should include sharing knowledge, skills, and effort among the individuals to reach the solution. CPS competency was assessed using a 4 x 3 matrix (OECD, 2017). In the matrix, the row heading represented the individual problem-solving skills (exploring and understanding, representing and formulating, planning and executing, monitoring and reflecting). The column heading represented the collaboration skills (establishing and maintaining shared understanding, taking appropriate action to solve the problem, and establishing and maintaining team organization).

Unlike PISA, ATC21S defined CPS more generally as "approaching a problem responsively by working together and exchanging ideas" (Griffin & Care, 2015, p. 38). In this sense, ATC21S saw CPS as both an activity of solving problems together and competence to approach that problem in a group. Like PISA, ATC21S considered collaboration as different from cooperation, i.e., cooperation is an activity being accomplished by dividing the work needed; hence, the individuals involved work on it in parallel. On the other hand, in CPS, the activities were constructed interchangeably among individuals, and others may continue one work. This definition affects how ATC21S constructs its CPS assessment framework by designing a task that requires collaboration instead of cooperation.

The ATC21S framework for assessing CPS skills was based on two main skills: social skill, which referred to collaboration, and cognitive skill, which referred to problem-solving (Griffin & Care, 2015). Social skills included participation, perspective-taking, and social regulation, while cognitive skills included task regulation, learning, and knowledge building. The framework for problem-solving skills utilized the theory of Polya (1945) and OECD PISA. The framework of problem-solving skills in both frameworks could be traced back to four steps of problem-solving skills identified by Polya, i.e., see, plan, do, and check; although in the ATC21S framework, the indicators were more specified in each. The "participation", "perspective taking", and "social regulation" sub-dimensions in the ATC21S framework had much in common with PISA's "Taking appropriate action to solve the problem", "Establishing and maintaining shared understanding", and "Establishing and maintaining team organization" respectively.

The main articles, which focused on collaborative problem-solving in mathematics, positioned CPS as a situation where students were investigated. Those studies' theoretical reviews addressing CPS described CPS as an activity of solving problems together. However, a certain quality of activities was identified using an indicator similar to that of ATC21S and PISA. For example, a study by Mercier et al. (2017) incorporated the theory of perspective-taking by a member of the group to identify a successful interaction of CPS in mathematics classrooms. Another example was the study by Taylor and McDonald (2007), who identified students' problem-solving skills in a group using the framework by Polya similar to that of PISA. Rather than seeing CPS as a whole competency, the main articles perceived CPS as an activity where the study's focus happened or was investigated. Participants in those studies were asked to solve a mathematical problem or do a certain mathematical task in a group. Participants' interaction or performance was recorded and analyzed during the ongoing problem-solving process. In those studies, CPS was the environment or situation where participants solved a problem or task together, and the task or problem was designed for study focus.

Task characteristics

CPS task characteristics in mathematics are that it is a mathematics problem-solving task (which expects students to use their problem-solving skills) but being put in a collaborative setting. By that, the task or the problem should encourage students to share ideas and effort to get the solution. The use of open-ended or contextual problems attracted discussions and talks as they brought varied interpretations (Harding et al., 2017). Another example of a task was when a problem's information was divided into separated parts, like a jigsaw reading of a problem, and each part was given to each student (Griffin & Care, 2015; OECD, 2017). This setting could make the students share what they had since they need complete information to solve the problem. Shared understanding and effort were characteristics of CPS activity. Thus, CPS tasks in mathematics should provide students with a shared working space (all studies in the main articles had applied this). The shared space would act as the place to pool ideas and monitor them. For that purpose, the shared space should be accessible to all students and allow them to access their peers' work.

The task characteristics presented in the main articles were identified (see Table 3). One study might have more than one type of task; for example, a task might be both non-routine and in the form of a word problem. The definition of non-routine problems here followed that of Boonen et al. (2016), i.e., challenging problems whose no convenient model or solution path is readily available to apply. It was not surprising that most studies of the main articles utilized non-routine problems. Five studies utilized word problems, i.e., problems set in daily context (Verschaffel et al., 2010), and considering their complexity, none of them was a routine word problem. Diagram construction problems were identified from two studies on geometry topics and a study about linear function. The studies emphasized the construction problem to drive students' communication as well as explore students' complex visualization. The shared construction space was also the consideration in the diagram construction task.

Table 3
Task Characteristics in the Main Articles

Study	Types of tasks						
	Word problem	**Diagram construction**	**Jigsaw reading**	**Open-ended problem**	**Routine problem**	**Non-routine problem**	**Not specified**
(Chiu & Khoo, 2003)	√					√	
(Chiu, 2008b)	√					√	
(Dewi et al., 2019)							√
(Frings, 2011)	√					√	
(Granberg & Olsson, 2015)		√			√		
(Hurme et al., 2005)		√					
(Mercier et al., 2017)	√		√			√	
(Munson, 2019)					√		
(Oner, 2013)		√				√	
(Schmitz & Winskel, 2008)						√	
(Tatsis & Koleza, 2008)						√	
(Taylor & McDonald, 2007)	√			√		√	
(Utami & Hwang, 2021)					√		

The fact that only one jigsaw reading of a problem and open-ended problem were utilized by the studies needed to be underlined. The jigsaw problem, where each student had a part of it and had to share it with team members, was utilized by Mercier et al. (2017). The problem was designed to increase the possibility of interaction among group members. As the study focused on collaborative interaction, jigsaw reading of the problem might be the best type of task to support it. A similar reason was for the study by Taylor and McDonald (2007) that utilized open-ended problems, i.e., problems with multiple possible solutions (Chan & Clarke, 2017). The study used this type to broaden the writings possibility to accommodate its focus on investigating a writing heuristic for CPS. The fact that three studies utilized routine problems might be due to their less reliance on the complexity of students' CPS process or the mathematical challenge students face. Despite being different in some ways, there were common characteristics for the CPS task held by most of the main articles, i.e., the task should be mathematically complex, stimulate sharing of ideas, and the availability of shared working space.

Themes of studies on CPS in mathematics

An analysis was conducted to identify the themes and research design of the selected articles. The methods and findings of the thirteen studies were examined individually to develop an appropriate theme, and shared themes were considered. In this process, five themes were identified: Technology, Discourse analysis, Social Skills, Mathematical thinking, and Students' achievement (see Table 4).

Table 4
Themes of the Main Articles

Study	Theme found				
	Technology	**Discourse analysis**	**Social Skills**	**Math thinking**	**Students' achievement**
(Chiu & Khoo, 2003)		√			√
(Chiu, 2008b)		√			√
(Dewi et al., 2019)			√		
(Frings, 2011)				√	
(Granberg & Olsson, 2015)	√			√	
(Hurme et al., 2005)	√			√	
(Mercier et al., 2017)	√				
(Munson, 2019)		√			
(Oner, 2013)	√				
(Schmitz & Winskel, 2008)		√			
(Tatsis & Koleza, 2008)		√	√		
(Taylor & McDonald, 2007)			√		√
(Utami & Hwang, 2021)	√				√

A technology theme refers to studies that investigate particular technology-based tools or media as the aim. The discourse analysis theme refers to studies that mainly use the discourse as data and extract findings mostly from it. For comparison, some studies on the Social skills theme did analyze utterances but used them to support the analysis of certain social aspects as the study focus (such as communication or norms). Studies in mathematical thinking investigated particular thinking such as reasoning or metacognition. Students' achievement theme was given to studies investigating the effect of certain treatment or media on students' performances. Each theme and future recommendations were described.

The use of technology for CPS

The first theme is technology. All five studies in this theme show the benefits of a particular technology for students' collaborative problem-solving. The study by Hurme and Jarvela (2005), Granberg and Olsson (2015), and Oner (2013) found how the media gave space for students to visualize their thinking during CPS activities. In the first study, students solve math problems on a network-based platform. The platform supported the group discussion because students could monitor their friends' thinking and make judgments directly on the platform. Similar benefits were provided by Geogebra, the technology being investigated in the second study. Students in groups could give direct feedback to their peers in the shared workspace and engaged in an argumentation. Considering the needs of visualization demanded from geometry, the third study described the use of Geometry Sketchpad for their collaborative discussion.

The other two studies analyzed the effectiveness of a particular technology by comparing it with non-digital media. Mercier et al. (2017) showed that students engaged more in CPS using a multi-touch tool than paper. The higher engagement was shown through students' efforts to elaborate or combine ideas. Another study by Utami and Hwang (2021) built a "U-decimal" application in teaching. It showed that it effectively improved students' group problem-solving achievement compared to traditional teaching.

It could be summarized that studies of the use of technology concluded its benefits to CPS either as tools to support communication among students or to help them engage with the mathematics content. However, these studies were conducted in a controlled or designed environment. Thus, they recommended the investigation of using technology in a more naturalistic way (e.g., using the content of the curriculum, in the real classroom, etc.), which was discussed in some other literature (e.g., Sanchez, 2020). In addition, further studies should address the technical limitation brought by the technology.

Discourse analysis for investigating CPS

The studies emphasized the analysis of utterances and conversations made during CPS activities in the discourse analysis theme. A study by Chiu and Khoo (2003) analyzed what factors affect the correct solution to a problem during CPS. They coded types of speakers' turns as variables such as disagreement, rudeness, justification, etc., and used statistical analysis to identify which one positively affected correct contribution. Further investigation by incorporating qualitative analysis for the same issue was done by Chiu (2008b). It analyzed types of utterances made by participants that would increase the possibility of correct contribution to the problem solving, i.e., wrong contribution, correct evaluation of ideas, justification, and polite disagreement.

A study by Tatsis and Koleza (2008) analyzed social and socio-mathematical norms established during CPS activities through utterances. The utterances made by participants were coded, and it turned out that nine norms were being identified. Similar to it, by coding the keywords made by students, a study by Schmitz and Winskel (2008) revealed that groups with different abilities used different keywords during CPS. Quite differently from the first four, a study by Munson (2019) analyzed the teachers' discourse and what prompts they made to guide CPS activities. The study found five distinctive pathways made by teachers, i.e., praise, nudge, attempted nudge, funnel, and switching continuously from attempted nudge to funnel.

In all five studies on this theme, discourse analysis was used to identify different types of utterances during CPS. The codes extracted from the utterances were analyzed to test the hypotheses quantitatively or to describe the issue qualitatively. All studies on this theme recommended future research investigating how different types of participants (by gender, native language, and social context) might give different results. Two studies (Chiu, 2008b; Tatsis & Koleza, 2008) recommended future studies on how the use of formal mathematics language compared to daily language may affect the process of CPS.

Social skills during CPS

The social skills theme comprised three studies that highlighted communication, collaboration skills, and social interaction. The study by Taylor and McDonald (2007) found that communication skills were being improved through writing instructions in CPS activities. The writing process helped students regulate their thinking and exercise formal mathematical communication. Instead of as a benefit, a study by Tatsis and Koleza (2008) found collaboration skills as a factor in promoting successful CPS. It was obvious in the study when participants did not put enough effort into reaching a mutual agreement and sharing ideas; the problem-solving process was hindered. A study by Dewi et al. (2019) identified types of interaction between different groups of students and teachers during CPS. High-ability groups showed better quality of discussion by having three types of interaction: students to students, students to teacher, and students to materials.

All studies on this theme identified a particular social strand (i.e., communication skill, collaboration skill, or social interaction) in the context of CPS. The studies mentioned discussed heavily how the context of participants might affect the finding interpretation of the social strand quality, showing that the CPS process was highly intertwined with social context. Both studies were context-dependent, the studies recommended future investigations of the different contexts of participants or analyzing how the social context related to the CPS process or performance.

Mathematical thinking during CPS

Mathematics thinking themes contained three studies by Hurme and Jarvela (2005), Grandberg and Olsson (2015), and Frings (2011). The first study analyzed how students used metacognitive knowledge and made metacognitive judgments during CPS activities. It was interesting that the metacognitive activities were apparent through group discussion and that participants regulated not only their thinking but also suggested peer's thinking. The second study investigated creative reasoning used by students in CPS activities. The creative reasoning was analyzed through students' arguments during problem-solving, and the ideas were shown through students' trial-and-error.

Quite different from the first two, a study by Frings (2011) investigated fatigued adults' performance in solving mathematics problems from the perspective of cognitive flexibility. It was shown that solving problems collaboratively did not increase *Einstellung* (low cognitive flexibility), while the ones who did it individually showed an increased *Einstellung*. The study enriched our understanding that CPS might lessen cognitive load compared to individual problem-solving. All studies on this theme used CPS as a situation where a certain construct of mathematical thinking was investigated. The studies suggested investigating what kinds of discussion would support students' mathematical thinking in the future. Investigation on how or to what extent particular mathematical thinking affected or promoted CPS performance was still scarcely represented.

Students' CPS achievement

Four studies quantitatively investigated students' achievement in CPS and belonged to the students' achievement theme. Two proved the effectiveness of a certain treatment in improving students' test scores. A study by Taylor and McDonald (2007) proved that writing increased group scores on CPS activities, while a study by Utami and Hwang (2021) proved that an experimental group using an application, namely *U-decimal* scored significantly higher in CPS than the one who did not. In the other two studies, Chiu and Khoo (2003) revealed that successful group agreement on a certain idea or answer was affected positively by academic status and negatively by justifications, and score on CPS was affected positively by academic grade and negatively by rudeness. Chiu (2008) used dynamic multilevel analysis to identify what type of conversation led to successful CPS and measured its effect. The study revealed that wrong contribution, correct evaluation of ideas, justification, and polite disagreement increased the possibility of successful CPS, with justification having the greatest effect.

All studies in this theme analyzed what factors or tools could positively affect students' CPS performance or success. In this case, a quantitative analysis of data collected from the sample was conducted. These studies recommended using a larger sample and longer treatment periods for better analysis.

Three studies quantitatively investigated students' achievement in CPS belonged to the students' achievement theme. Two proved the effectiveness of a certain treatment to improve students' test scores. A study by Taylor and McDonald (2007) proved that writing increased group scores on CPS activities, while a study by Utami and Hwang (2021) proved that an experimental group using an application, namely U-decimal scored significantly higher in CPS than the one who did not. In the last study, Chiu (2008) used dynamic multilevel analysis to identify what type of conversation led to successful CPS and measured its effect. The study revealed that wrong contribution, correct evaluation of ideas, justification, and polite disagreement increased the possibility of successful CPS, with justification having the greatest effect. All studies on this theme recommended using a larger sample and longer treatment periods. The summary of findings for the thirteen selected articles can be seen in the Appendix.

Conclusion, Limitations, and Future Research

In conclusion, the systematic literature review of the thirteen articles discussed the definition, task characteristics, and some themes of studies of CPS in Mathematics. CPS could be an activity where individuals solve problems together or as a problem-solving competency in groups. CPS could be measured as a competency by analyzing mathematical problem-solving and collaborative skills. When being perceived as an activity, CPS required some characteristics of the task, i.e., a complex problem, stimulating sharing of ideas, and the existence of a shared working space.

Five themes of studies on CPS in mathematics were identified, i.e., technology, discourse analysis, social skills, math thinking, and students' achievement. The themes informed the current knowledge learned from studies in CPS in mathematics. Some future recommendations could be summarized from the review for each theme. Related to technology, studying the use of certain technology in a naturalistic environment (not a controlled, designed environment) can be studied further. Investigating the CPS of students based on their personal or social background was also recommended. For instance, studying CPS of mixed gender compared to the same gender in a group or studying how different language backgrounds affect CPS performances. Some studies recommended investigating mathematical thinking deeper during CPS activities and how it supports or is supported by CPS. For more quantitative analysis, a study using large samples might better generalize what has been learned from previous studies.

This study has some limitations. The title search used only the Scopus database; thus, the findings may not represent studies outside the Scopus database. As collaborative problem solving can also be represented by other terms, the search keywords used in this study might not capture all studies involving CPS. Therefore, it is recommended to use other databases and other relevant keywords in the future to capture the issues around CPS in mathematics.

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Appendix

The main articles' summary, sorted by authors' name (quan: quantitative; qual: qualitative).

No	Title	Author(s)	Aim	Level	Method	Results
1	Rudeness and Status Effects During Group Problem Solving: Do They Bias Evaluations and Reduce the Likelihood of Correct Solutions?	(Chiu & Khoo, 2003)	Tested whether several factors significantly influenced how group members evaluated the correctness of one another's ideas and whether these factors affected GPS success.	9th	Quan: Eighty high school students completed a questionnaire and were videotaped doing an algebra problem in groups of 4. A new statistical method for analyzing group processes was used to analyze agreement and solution score predictors.	The previous speaker's correctness predicted agreement. In successful groups, the agreement was also affected positively by academic status and negatively by justifications. In unsuccessful groups, however, an agreement was affected positively by recent agreements and negatively by recent rude actions. Solution score was predicted positively by academic grade, and percentage of right turns but negatively by rudeness.
2	Flowing toward correct contributions during group problem solving: A statistical discourse analysis	(Chiu, 2008b)	Investigated elements affecting correct contribution (CC) in CPS.	High school	Quan: dynamic multilevel path analysis (DMA) identified breakpoints that divided conversation into periods with many CCs vs. few CCs and created the model. Qual: transcript analysis to support quantitative results	Wrong contribution, correct evaluation of ideas, justification, and polite disagreement increased the possibility of CC. Questions, rude disagreement, and agreement reduced it. Justification had the biggest effect.
3	The interaction of students in mathematical problem solving with group discussion activities	(Dewi et al., 2019)	Identified and described the types of interactions that arise in learning when students solve math problems through group discussion activities.	4th	Qual: Observations were made of three groups, i.e., high, medium, and low group abilities. Formation	Three types of interactions for high groups, i.e., students – students, students – teachers, and students – material; two types of interactions for the medium group, i.e., students – teachers, and students – material; two types of interactions for low groups, i.e., students – students and students – material.
4	The effects of group monitoring on fatigue-related *Einstellung* during mathematical problem solving	(Frings, 2011)	Tested whether fatigue leads to increased *Einstellung* (low levels of cognitive flexibility) in a series of mathematical problem-solving tasks	Adults	Quan: participants attempted to solve a series of problems either alone or in a team, and while either reasonably alert (non-fatigued) or fatigued through sleep deficit	Fatigued problem solvers working alone showed increased *Einstellung*. In contrast, teams of fatigued problem solvers did not experience increased Einstellung.

No	Title	Author(s)	Aim	Level	Method	Results
5	ICT-supported problem solving and collaborative creative reasoning: Exploring linear functions using dynamic mathematics software	(Granberg & Olsson, 2015)	Investigated how GeoGebra may support students' collaboration and creative reasoning during mathematical problem solving	High school	Qual: students worked in pairs to solve a linear function problem using Geogebra. Recorded conversation and computer analysis were analyzed	Geogebra supported collaboration and creative reasoning by providing a shared workspace and feedback. Some students used GeoGebra for trial-and-error strategies, and students engaged in superficial argumentation
6	Students' activity in computer-supported collaborative problem solving in mathematics	(Hurme et al., 2005)	Analyzed students' computer-supported collaborative mathematical problem solving, especially the metacognitive process and the applicability of network-based environment	High school	Qual: Content-analysis of the computer notes, PS activities, and metacognitive activities	Networked discussions mediated mathematical knowledge and students' questions. The students co- regulated their thinking, used metacognitive knowledge, made metacognitive judgments, and performed monitoring during networked discussions.
7	Student interactions and the development of ideas in multi-touch and paper-based collaborative mathematical problem solving	(Mercier et al., 2017)	Compared the interactions of groups of 10 to 11-year-old students working collaboratively to solve three math problems in either a multi-touch or paper condition	High school	Qual: analysis of students' ideas, who responded to them, and how they were responded to identify idea development and interactions. Quan: descriptively to visualize the spread of data.	Higher engagement in multi-touch than paper. Student responses to ideas raised by other students were more likely to elaborate on the idea or combine it with other ideas in the multi-touch condition than in the paper condition.
8	After eliciting: Variation in elementary mathematics teachers' discursive pathways during collaborative problem solving	(Munson, 2019)	Explored the discursive pathways two fourth-grade mathematics teachers used after eliciting student thinking when they aimed to be responsive to and advance student thinking	Teacher	Qual: analyze nine lessons from two teachers (97 interactions) to identify distinctive pathways	Five distinctive pathways made by teachers: praise, nudge, attempted nudge, funnel, attempted nudge–funnel
9	Analyzing group coordination when solving geometry problems with dynamic geometry software	(Oner, 2013)	Proposed a methodological approach for studying coordination of resources when solving geometry problems with dynamic geometry software	University	Qual: The interaction between the students was captured through two video cameras. The data were transcribed using the notations of conversation analysis.	The participants mainly took advantage of two types of resources within their interaction: collaborative (relevant for the relational or social aspect) and mathematical and GSP-enabled resources (relevant for the content-related aspect).

No	Title	Author(s)	Aim	Level	Method	Results
10	Towards effective partnerships in a collaborative problem-solving task	(Schmitz & Winskel, 2008)	Investigated the effect of pairing students with different abilities (high-low and medium-low) and their partner's being helpful or unhelpful to the quality of conversation when solving problems	6th	Quan: Comparing the number of keywords that appeared between 4 groups (high-low with helping role and not, medium-low with helping role and not) quantitatively (ANOVA). Qual: analyzed student conversation transcripts and formulated interaction patterns.	Quan: significant differences were found between groups with different abilities but not among groups with different roles Qual: the group with good conversational quality is medium-low with the help of roles. Good conversation quality is characterized by exploration of members' ideas
11	Social and socio-mathematical norms in collaborative problem-solving	(Tatsis & Koleza, 2008)	Investigated how social and socio-mathematical norms are established during the interactions of CPS	University	Qual: analysis of the recorded video and transcripts. The effect of the establishment of norms was also analyzed qualitatively	Norms are found, especially related to particular aspects of the problems. Most norms enhanced the PS process. Lack of collaboration norms leads to unsuccessful PS
12	Writing in groups as a tool for non-routine problem solving in first year university mathematics	(Taylor & McDonald, 2007)	Checked if the writing heuristics were helpful for students in CPS	University	Qual: Polya's problem-solving framework and a writing heuristic are used synergistically with group work to develop the mathematical skills and enhance the mathematical communication skills of novice students Quan: score on tests were analyzed and represented into boxplots (before-after using writing)	Writing helped students regulate their thinking and boost communication. Group work while writing was proven effective for problem-solving
13	The impact of collaborative problem posing and solving with ubiquitous-decimal app in authentic contexts on math learning	(Utami & Hwang, 2021)	Investigated the effect of student's learning behaviors in CPPS activity supported by U-Decimal on their decimal learning achievement	4th	Quant: treatment group with learning app "U-decimal," control group without	(1) the experimental students scored significantly higher on the post-test than the control students. (2) the contributing functional aspects (i.e., picture-taking function, multimedia- text and voice explanation, and peer assessment) and behavioral factors that predict learning achievement (i.e., quality of multiple-choice question design, quality of comment, and explaining answer using multimedia-text and voice explanation).

Problemų sprendimas bendradarbiaujant matematikos srityje: sisteminė literatūros apžvalga

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Santrauka

Reaguojant į XXI amžiaus pokyčius atlikta daugybė problemų sprendimo bendradarbiaujant (angl. *CPS*) tyrimų matematikos srityje. Šio tyrimo tikslas – ištirti šių problemų sprendimo apibrėžtį ir užduočių ypatumus bei nustatyti tyrimų temas. Naudojantis Scopus duomenų baze buvo atlikta sisteminė literatūros apžvalga (angl. *SLR*), buvo analizuojami 2002–2022 m. publikuoti straipsniai. Atlikus peržiūrą iš 293 straipsnių atrinkta 13 straipsnių. Remiantis atrinktais straipsniais, problemų sprendimą bendradarbiaujant matematikos srityje galima apibrėžti kaip veiklą, kurios metu asmenys kartu sprendžia problemas, arba kaip problemų sprendimo kompetencijos įgijimą bendradarbiaujant grupėse. Suvokiant problemų sprendimą kaip veiklą, reikia tam tikrų matematikos užduoties pateikimo savybių, t. y. sudėtingų problemų sprendimų, skatinančių dalytis idėjomis, ir bendros darbo erdvės egzistavimo. Problemų sprendimas bendradarbiaujant matematikoje gali būti suvokiamas ir kaip kompetencija, kurią sudaro problemų sprendimo ir bendradarbiavimo įgūdžiai matematikoje.

Teminė analizė leido išskirti penkias pagrindines tyrimų temas: technologijos, diskurso analizė, socialiniai įgūdžiai, matematinis mąstymas ir mokinių pasiekimai. Išnagrinėtos ir aprašytos kiekvienos temos tyrimų spragos ir galimi bei būsimi tyrimai. Šiuo tyrimu prisidedama prie dabartinio supratimo apie problemų sprendimą matematikos srityje. Rekomenduojama naudoti daugiau straipsnių, kad būtų galima išsamiau aprašyti susijusias problemas.

Esminiai žodžiai: *problemų sprendimas bendradarbiaujant, matematikos mokymas, sisteminė literatūros apžvalga, tendencijos.*

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